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Quasi-Static Manipulation with Hemispherical Soft Fingertip via Two Rotational Fingers

Takahiro Inoue and Shinichi Hirai
Department of Robotics, Ritsumeikan Univ.,
1-1-1 Noji-Higashi, Kusatsu, Shiga 525-8577, Japan
gr018026@se.ritsumei.ac.jp

***Abstract*— We derive a local minimum of an elastic potential energy due to the deformation of a hemispherical soft fingertip, and propose a quasi-static manipulation algorithm using the local minimum of the potential energy by means of two rotational fingers, on which the soft fingertips are mounted. In this model, a geometrical constraint between the grasped object and two fingertips, which includes the deformation of the fingertip, is derived. Using the constraint and the local minimum of the potential energy, we newly propose a numerical algorithm in the quasi-static manipulation. Finally, we conclude that the existence of the local minimum allows us to stably grasp an object in soft fingered manipulation.**

***Index Terms*— Soft fingertip, Manipulation, Elastic force, Local minimum.**

I. Introduction

Almost all conventional researches focusing on the manipulation process of a grasped object have assumed a point-contact between the object and a rigid fingertip. Since the grasped object moves and rolls on the fingertip without any change of the radius of the fingertip in the case of the rigid point-contact model, it is comparatively easy to analytically describe position and velocity equations based on a geometrical relationship, which are required for the stability analysis of the grasped object. On the contrary, soft fingertips tend to deform easily and largely due to their softness. In fact, since a plane-contact occurs during soft fingered manipulation, the contact point cannot be determined uniquely. Furthermore, The plane-contact is necessary for humans to achieve stable grasping and manipulation of the object, because the elastic force acting on the soft fingertip disperses widely to the object.

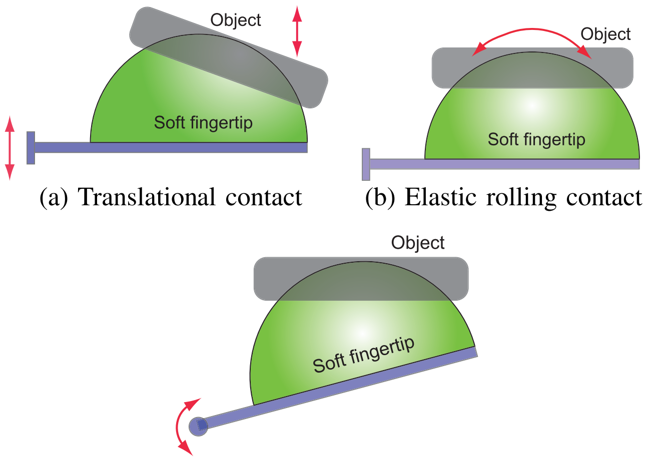
In terms of above, recently several researches associated with the grasping using a soft fingertip have been studied. Xydas and Kao *et al.* [1]–[3] have shown an exact deformation shape of a hemispherical soft fingertip by using FEM analysis. These above studies, however, have focused only on deriving the more exact deformation model, and have proposed only the vertical contact deformation of the soft fingertip. In addition, that paper have not mentioned any equations of motion of the grasped object, which are required for the analysis of manipulation stability. Nguyen *et al.* [4] have proposed a simple deformation model of a soft fingertip in order to use analytical mechanics theory in control. The deformation model, however, assumes that all the elastic forces acting on the soft fingertip face toward the origin of the fingertip. Therefore, it is difficult to represent the contact model of the fingertip and equations of motion of the object relating to the plane-contact. Arimoto *et al.* [5] and Doulgeri *et al.* [6], [7] have proposed a control law for pinching motion of a grasped object using soft fingertips, which needs a superfluous control input to control an elastic rolling motion of the object due to the lack of a concept of local minimum of elastic potential energy caused by deformation of the soft fingertip. Inoue *et al.* [8], [9] have proposed three contact models between a soft fingertip and a rigid object: translational contact, rotational contact, and elastic rolling contact models, and formulated elastic force equations that appear as repulsive forces against the grasped object.

In this paper, we redescribe our translational contact model shortly and represent a deriving process of the local minimum of the elastic potential energy due to the deformation of the soft fingertip. Furthermore, we newly propose a numerical algorithm in a quasi-static manipulation using a concept of the local minimum of the potential energy, and simulate our model to compare the motion of the grasped object with an experimental result.

II. Proposed Three Models and Elastic Forces

In this study, all the possible contact patterns between the object and the fingertip are categorized into three patterns according to the motion of a finger, the posture of an object, and the direction of an applied force: translational contact, rotational contact, and elastic rolling contact, as shown in Fig.1. In each contact model, the infinite number of virtual springs are introduced along vertical direction, whose individual spring constants are different for each other due to their natural lengths that stem from the hemispherical shape of the fingertip, as shown in Fig.2 [8].

As shown in Fig.3, let d be the maximum displacement, and a be the radius of the fingertip. Let Σ_{fi} be the fingertip coordinate system, O_{fi} be the origin of Σ_{fi} coordinate system, and R be the arbitrary point on xy-plane of Σ_{fi} coordinate system. Let us introduce a virtual spring on the point R perpendicular to xy-plane, which has an infinitesimal sectional area dS. Let Q and P be the upper ends of the spring in a natural state and after deformation, respectively. The infinitesimal elastic forces that act on the shrank part of the single virtual spring are respectively described as



(a) Translational contact (b) Elastic rolling contact

(c) Rotational contact

Fig. 1. Main three contact patterns

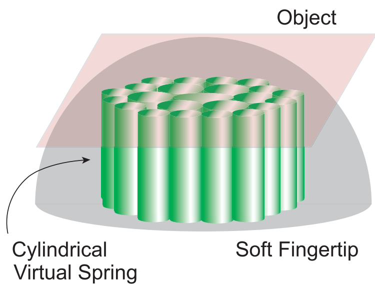


Fig. 2. Cylindrical virtual spring components

follows [8]:

$$dF(x,y,d,\theta_{obj}) = E\varepsilon(x,y,d,\theta_{obj})dS, \quad (1)$$

where

$$\varepsilon(x,y,d,\theta_{obj}) = 1 - \frac{a-d-x\cdot\sin\theta_{obj}}{\cos\theta_{obj}\sqrt{a^2-(x^2+y^2)}}. \quad (2)$$

Note that the symbol E means Young's modulus and eq.(2) indicates a physical quantity that corresponds to a strain. Since eq.(1) satisfies Hooke's law, the derived force equation is based on mechanics of materials. Furthermore, the symbol $dF(x,y,d,\theta_{obj})$ represents that the infinitesimal elastic force is a function of four variables x, y, d, and θ_{obj}.

III. Local Minimum of Potential Energy

In our latest study, we have found a local minimum of an elastic potential energy generated on the hemispherical soft fingertip due to its deformation.

Let us formulate an elastic potential energy of the shrank part of the fingertip from the translational contact model represented as eq.(1). Let k be the virtual spring constant and PQ be the shrinkage of the single virtual spring QR [8]. The infinitesimal elastic potential energy due to the shrinkage PQ is then described as follows:

$$dU(x,y,d,\theta_{obj}) = \frac{1}{2}k\cdot PQ^2 = \frac{1}{2}E\lambda(x,y,d,\theta_{obj})dS, \quad (3)$$

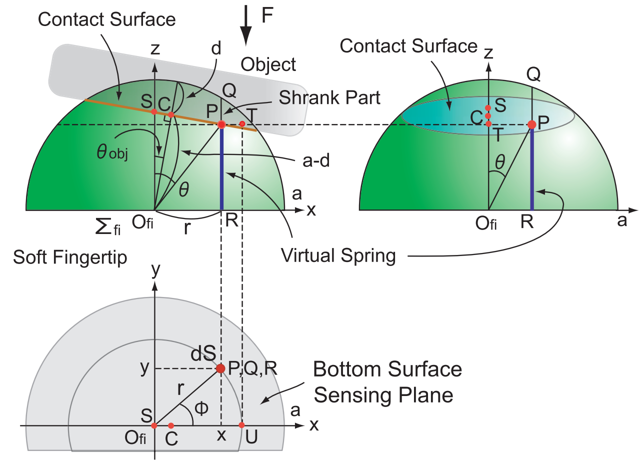


Fig. 3. Translational contact

where

$$k = \frac{EdS}{\sqrt{a^2-(x^2+y^2)}}, \quad (4)$$

$$\lambda(x,y,d,\theta_{obj}) = \sqrt{a^2-(x^2+y^2)} - 2\frac{a-d-x\cdot\sin\theta_{obj}}{\cos\theta_{obj}} + \frac{(a-d-x\cdot\sin\theta_{obj})^2}{\cos^2\theta_{obj}\sqrt{a^2-(x^2+y^2)}}. \quad (5)$$

Therefore, performing the double integration within the elliptic integration area shown in Fig.4 with respect to x and y, the total elastic potential energy can finally be written by

$$U(d,\theta_{obj}) = \frac{1}{2}E\int_A^B\int_{-b(x)}^{b(x)}\lambda(x,y,d,\theta_{obj})dydx, \quad (6)$$

where

$$A = (a-d)\sin\theta_{obj} - \sqrt{a^2-(a-d)^2}\cos\theta_{obj}, \quad (7)$$

$$B = (a-d)\sin\theta_{obj} + \sqrt{a^2-(a-d)^2}\cos\theta_{obj}, \quad (8)$$

$$b(x) = \sqrt{a^2-(a-d)^2-\frac{\{x-(a-d)\sin\theta_{obj}\}^2}{\cos^2\theta_{obj}}}. \quad (9)$$

Next, let us represent a local minimum of the elastic potential energy due to the deformation of a soft fingertip. Since it is difficult to calculate the analytical double integration of the potential energy equation, eq.(6), we specify the local minimum of the potential energy by performing a numerical analysis. Fig.5 shows simulation results, in which the object orientation varies from -25deg to 25deg. Four lines correspond to different maximum displacements: 2.0mm, 4.0mm, 6.0mm, and 8.0mm. The vertical axis denotes an elastic potential energy due to the entire deformed part of the fingertip. In this simulation,

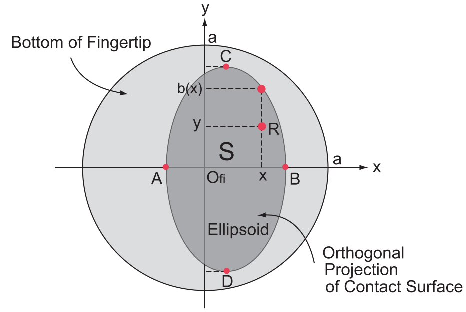


Fig. 4. Integration area

the radius of the soft fingertip is decided as 20mm that is equal to the radius of an actual soft fingertip fabricated for experiment.

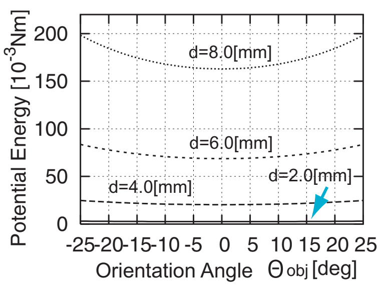


Fig. 5. Local minimum of potential energy

This result shows an existence of the local minimum of the elastic potential enegy when the orientation angle is in 0deg at every curve. The depth of the potential energy increases gradually as the maximum displacement rises to 8mm. This means that the grasped object by two fingered hand may quickly converge to an equilibrium point, which corresponds to $\theta_{obj} = 0$deg, when the maximum displacement is large in an actual manipulation process. This result indicates that the local minimum of the elastic potential energy will play a substantially important role in realizing stable grasping and manipulation of an object with soft fingertips.

IV. Two Fingered Manipulation

To describe the motion of the grasped object, the position and the velocity of a contact point between the fingertip and the object should be clarified in the case of a rigid fingertip. Contrary, the geometrical relationship containing the deformation of the soft fingertip should be specified because the point contact could not be obtained in the soft fingered manipulation. In this section, we derive a geometrical constraint between both fingertips through the grasped object to represent the position and the motion of the object when the width of the object and the distance between both fingers are given.

A. Geometrical Constraint

As shown in Fig.6, let a be the radius of both fingertips, d_r and d_l be the maximum displacement of each fingertip, O_r and O_l be the origin of each fingertip, and R and U be the points of the root of the each finger. Let θ_{obj} be the orientation angle, and W_{obj} be the width of the grasped object. Furthermore, W_{fi} be the width between both fingers, and L be the length of the finger. Let Σ_R be the reference coordinate system, Σ_U be the right finger coordinate system, and its origin is identical to the point U. Coordinates of the point O_r with respect to Σ_U and the point O_l with respect to Σ_R are then represented as follows:

$$\vec{O_r} = \begin{bmatrix} -L\sin\theta_r \\ L\cos\theta_r \end{bmatrix}, \tag{10}$$

$$\vec{O_l} = \begin{bmatrix} L\sin\theta_l \\ L\cos\theta_l \end{bmatrix}. \tag{11}$$

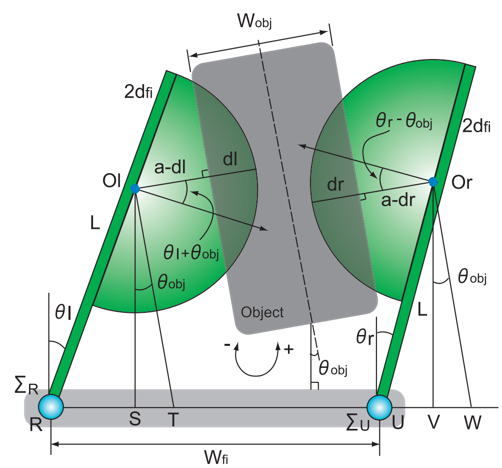


Fig. 6. Geometrical constraint

In addition, each point with respect to Σ_R coordinate system is then described as follows:

$$\vec{RS} = \begin{bmatrix} L\sin\theta_l \\ 0 \end{bmatrix}, \tag{12}$$

$$\vec{RT} = \begin{bmatrix} L\sin\theta_l + L\cos\theta_l \tan\theta_{obj} \\ 0 \end{bmatrix}, \tag{13}$$

$$\vec{RU} = \begin{bmatrix} W_{fi} \\ 0 \end{bmatrix}, \tag{14}$$

$$\vec{RV} = \begin{bmatrix} W_{fi} - L\sin\theta_r \\ 0 \end{bmatrix}, \tag{15}$$

$$\vec{RW} = \begin{bmatrix} W_{fi} - L\sin\theta_r + L\cos\theta_r \tan\theta_{obj} \\ 0 \end{bmatrix}, \tag{16}$$

where the thickness of the finger is negligible to avoid the complexity in the deriving process. The length TW can be described from eqs.(13) and (16) as follows:

$$\begin{aligned} TW &= W_{fi} - L(\sin\theta_r + \sin\theta_l) \\ &+ L\tan\theta_{obj}(\cos\theta_r - \cos\theta_l). \end{aligned} \quad (17)$$

The distance between the points O_r and O_l, which is along vertical direction to both lateral sides of the object, is written by

$$O_rO_l = W_{obj} + 2a - d_r - d_l. \quad (18)$$

The distance O_rO_l is rewritten from eq.(17) and the geometrical relationship shown in Fig.6 as follows:

$$O_rO_l = TW\cos\theta_{obj}. \quad (19)$$

Finally, calculating the above equation concretely, the maximum displacement d_l of the left fingertip is obtained as follows:

$$\begin{aligned} d_l &= W_{obj} + 2a - d_r - \{W_{fi} - L(\sin\theta_r + \sin\theta_l) \\ &+ L\tan\theta_{obj}(\cos\theta_r - \cos\theta_l)\}\cos\theta_{obj}, \end{aligned} \quad (20)$$

where the rotation angles θ_r and θ_l are given in the quasi-static manipulation, and W_{obj}, W_{fi}, and L are all constant parameters. Thus, once the object orientation θ_{obj} and the maximum displacement d_r of the right fingertip are given, we can calculate the maximum displacement d_l of the left fingertip by using eq.(20).

B. Position of Center of Gravity

Let us consider the position of center of gravity of the grasped object, as shown in Fig.7.

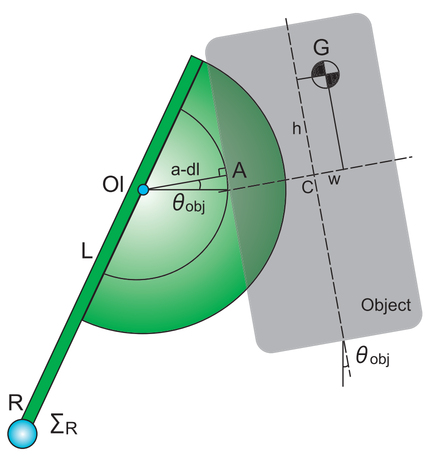


Fig. 7. Center of gravity

Let G be the center of gravity, w and h be the width and height from the center of the object respectively, and A be the foot of a perpendicular line from the point O_l to the object. The coordinate of the point A is then described, with respect to Σ_R coordinate system, using eq.(11) as follows:

$$\overrightarrow{RA} = \begin{bmatrix} L\sin\theta_l + (a - d_l)\cos\theta_{obj} \\ L\cos\theta_l + (a - d_l)\sin\theta_{obj} \end{bmatrix}. \quad (21)$$

Therefore, letting x_{obj} and y_{obj} be the position of center of gravity, the coordinate can finally be represented as below:

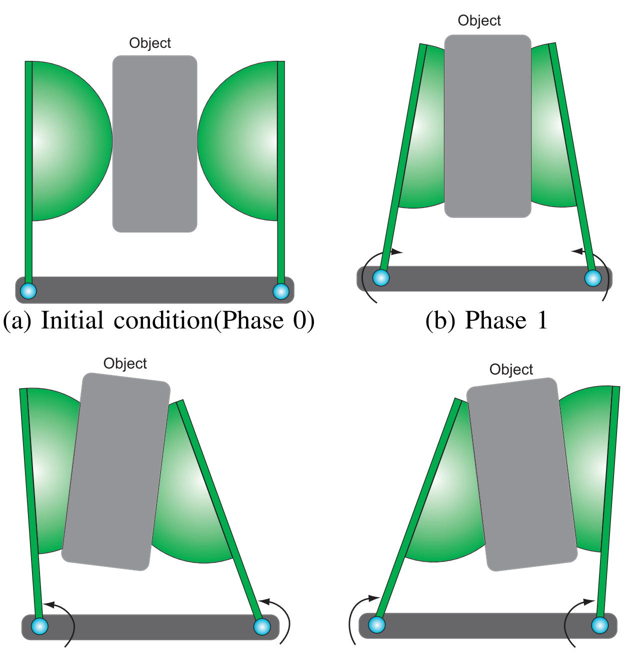
$$\begin{bmatrix} x_{obj} \\ y_{obj} \end{bmatrix} = \overrightarrow{RA} + \begin{bmatrix} \cos\theta_{obj} & -\sin\theta_{obj} \\ \sin\theta_{obj} & \cos\theta_{obj} \end{bmatrix} \begin{bmatrix} w + \frac{1}{2}W_{obj} \\ h \end{bmatrix}. \quad (22)$$

V. SIMULATION

We simulate our proposed model and show a quasi-static manipulation process based on the theory of the local minimum of the elastic potential energy of the soft fingertip.

A. Algorithm

As shown in Fig.8, we consider that two rotational fingers with hemispherical soft fingertips grasp a rigid object in an initial condition, in which we assume that the grasped object has planar surfaces in both sides and is a rectangular solid. Furthermore, the gravitational force is negligible in two dimentional plane.



(a) Initial condition(Phase 0) (b) Phase 1

(c) Phase 2 (d) Phase 3

Fig. 8. Simulation patterns

We have previously specified the existence of the potential energy due to the deformation of a hemispherical soft fingertip by introducing virtual springs along the vertical direction inside the fingertip. That is, the contact object moves and converges to an equilibrium point on the fingertip, which is equivalent to the parallel position between the object and the finger, as shown in Fig.3. The quasi-static algorithm in two fingered manipulation is that first each potential energy in both fingertips mounted on the right and left fingers is computed as well as the translational contact model when both fingers locate at given angles, as shown in Fig.6. Next, by summing up both potential energy, we compute a local minimum within the infinite possible patterns of the position and the posture of the object in the given condition of the fingers. Finally, we get three variables, x_{obj}, y_{obj}, and θ_{obj}, from the local minimum.

Let θ_{obj} and d_r be the variables that is necessary for representing the position and the posture of the grasped object because the maximum displacement of the left fingertip d_l is determined uniquely if d_r is once determined from eq.(20). Let U_r and U_l be the elastic potential energy of each fingertip, and $\theta_{obj,r}$ and $\theta_{obj,l}$ be the relative angles of the object with respect to Σ_{fi} coordinate system on each fingers as shown in Fig.3, repectively. As well as eq.(6), the total potential energy is then written by

$$U_{total} = \sum U_i(d_i, \theta_{obj,i}) \quad (i = r, l), \tag{23}$$

where

$$U_i(d_i, \theta_{obj,i}) = \frac{1}{2} E \int_{A_i}^{B_i} \int_{-b_i(x)}^{b_i(x)} \lambda(x, y, d_i, \theta_{obj,i}) \mathrm{d}y \mathrm{d}x. \tag{24}$$

Using the geometrical relationship shown in Fig.6, both relative angles $\theta_{obj,r}$ and $\theta_{obj,l}$ can be transformed as follows:

$$\theta_{obj,r} = \theta_r - \theta_{obj}, \tag{25}$$

$$\theta_{obj,l} = \theta_l + \theta_{obj}. \tag{26}$$

Since both rotational angles of the fingers θ_r and θ_l are given and d_l is described by using d_r in eq.(20), eq.(23) can be rewritten as follows:

$$U_{total} = U_r(d_r, \theta_{obj}) + U_l(d_r, \theta_{obj}) = U_{total}(d_r, \theta_{obj}), \tag{27}$$

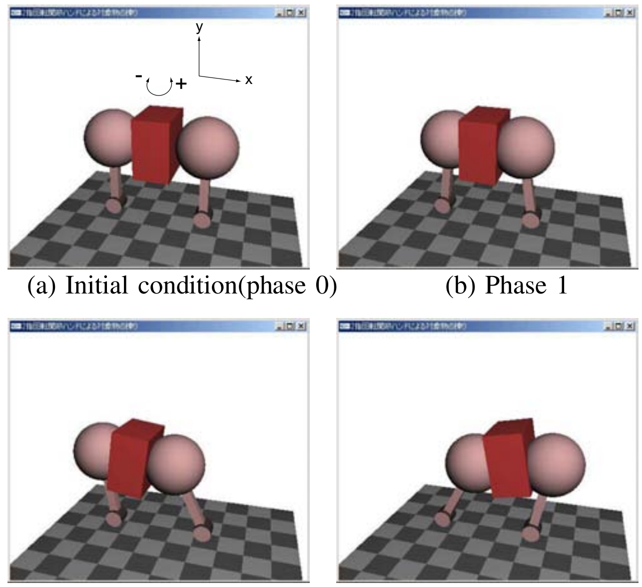
where the symbol (d_r, θ_{obj}) indicates that the potential energy U_{total} is a function of two variables of d_r and θ_{obj}. Finally, by obtaining two values d_r and θ_{obj} such that eq.(27) satisfies a local minimum of the potential energy in the numerical analysis, the position of the grasped object x_{obj} and x_{obj} can be calculated using eqs.(20), (21), and (22). Thus, the total potential energy is represented as a function of three variables: x_{obj}, y_{obj}, and θ_{obj}.

$$U_{total} = U_{total}(x_{obj}, y_{obj}, \theta_{obj}). \tag{28}$$

In the following section, we simulate the motion of the grasped object using this numerical algorithm such that x_{obj}, y_{obj}, and θ_{obj} are searched when the elastic potential energy takes a local minimum for every infinitesimal rotation angle of both fingers.

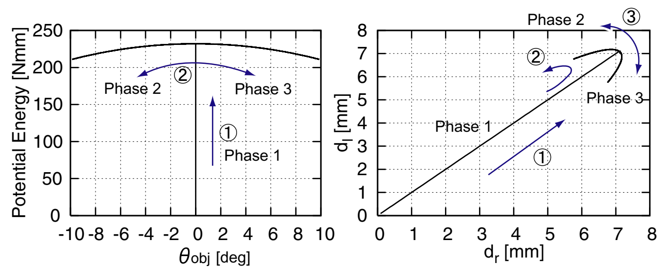
B. Simulation Results

In this simulation, we set that both fingers face each other in parallel in the initial condition, as shown in Fig.9-(a). We assume that Young's modulus, E, is 0.304MPa that is measured by a tension test of several specimens made of identical material with the soft fingertip, and its diameter is 40mm. The width and height, w and h, from the center of the object are zero to avoid the complexity. Also, the length of the finger, L, is set as 51mm, and the mass of the all parts is negligible in this simulation.



(a) Initial condition(phase 0) (b) Phase 1
(c) Phase 2 (d) Phase 3

Fig. 9. Motion of grasped object



(a) Potential energy (b) Both displacements

Fig. 10. Simulation results

As shown in Fig.9-(b) and Fig.10-(a), the total elastic potential energy by both fingers gradually increases as both fingers rotate toward inner side by 8deg, and the posture of the grasped object does not change. Continuously, when both fingers rotate toward counterclockwise direction by 20deg and rotate to opposite direction by 20deg, the object moves an opposite direction against the motion of both fingers. That is, the object slightly moves toward clockwise direction when both fingers rotate toward counterclockwise direction, and vice versa, as shown in Fig.9-(c) and (d). Additionally, after passed the maximum value shown in Fig.10-(a), the potential energy iterates a transition between maximum and minimum values, which corresponds to phase 2 and phase 3 shown in Fig.9. Furthermore, as shown in Fig.10-(b), each maximum displacement d_r and d_l of the right and left fingertips is completely equal at the case between phase 0 and phase 1. After that, d_l is larger than d_r in phase 2, and on the contrary, d_r becoms larger than

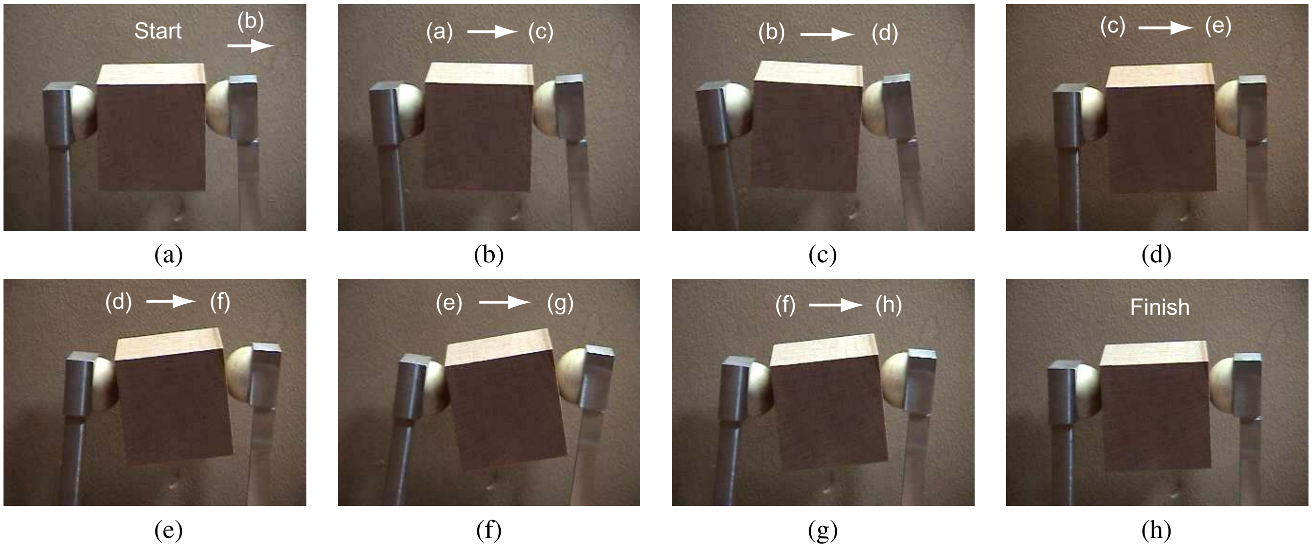


Fig. 11. Experimental results

d_l in phase 3.

Note that the position of center of gravity x_{obj} and y_{obj} and the orientation angle θ_{obj} can be decided while the total potential energy of both fingertips converges to a local minimum of eq.(28). Thus, we have obtained the quasi-static sequence motion of the grasped object by computing the variables x_{obj}, y_{obj}, and θ_{obj} consecutively at every infinitesimal angle of both fingers.

VI. Experiments

We evaluate the motion of the grasped object by two rotational fingers, and demonstrate that the object moves and tilts as well as the simulation results. We made a simple apparatus and a hemispherical soft fingertip, which is able to rotate freely by human hand motion as shown in Fig.11. The soft fingertip is made of polyurethane gel, and its diameter is 26mm. The length of each finger is 105mm, and the object is made of wood. Its shape is cube, and the mass is approximately 30g.

First, two rotational fingers are located at a parallel location, and rotate by a given angle toward inner side. The swing motion of both fingers is repeated several times. Fig.11 shows experimental results, in which the sequence motion of the object follows the arrows from (a) to (h). The grasped object moves from side to side as the fingers rotate simultaneously. The most important point is that the object tilts toward opposite direction of rotation of the fingers as well as the simulation result.

VII. Concluding Remarks

In this paper, we have newly proposed a local minimum of an elastic potential energy due to the deformation of a hemispherical soft fingertip, and mentioned an effectiveness of the local minimum and a numerical algorithm in terms of quasi-static manipulation by two rotational fingers. Furthermore, the motion and the posture of a grasped object based on our theoretical insight shown in simulation results have also found in experimental results. We conclude that the grasped object moves on the soft fingertip and converges to a determined position and posture such that the elastic potential energy satisfies a local minimum in every infinitesimal rotation of the fingers in the case of soft fingered manipulation. We infer that the local minimum of the potential energy plays a significant role in the stable manipulation even in human hands.

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