

# Robust Manipulation of Deformable Objects By A Simple Position Feedback

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Abstract

*Robust manipulation strategies of deformable objects will be presented. Manipulation of deformable objects can be found in many fields such as garment industry and food industry. Guidance of multiple points on a deformable object is a primitive operation in the manipulation of deformable objects. In this guidance, the points often cannot be manipulated directly. A model of the manipulated deformable object is needed in order to perform these operations. It is, however, difficult to build a precise model of a deformable object. Thus, we need a robust control scheme that allows us to realize the operations successfully despite of discrepancy between a manipulated deformable object and its model.*

*In this paper, we will firstly derive a mathematical model of deformable objects for their manipulation. Second, indirect simultaneous positioning operations of deformable objects are formulated. Then, we will propose a simple position feedback control law with the rough object model to realize the manipulation. The validity and the robustness of the proposed manipulation method will be shown through simulation results.*

## 1 Introduction

There exist many manipulative tasks that deal with deformable objects such as textile fabrics, rubber parts, paper sheets, and food products. Most these operations strongly depend on skilled human workers. We define manipulation of deformable objects as controlling of deformation of deformable objects as well as their positions and orientations in this paper. For example, a positioning operation called *linking* is involved in the manufacturing of seamless knitted products as shown in Fig.1 [1]. In linking of fabrics, knitted loops at the end of a fabric must be matched to those of another fabric so that the two fabrics can be sewed seamlessly. This operation is now done by skillful humans and automatic linking is required in manufacturing of knitted products. In this research, we describe the manipulations of deformable objects including linking by positioning of multiple points on the objects. Then, we regard the manipulations as the operations in which multiple points on a deformable object should be guided to the final locations simultaneously as

shown in Fig.2. In many cases these points cannot be, however, manipulated directly. Thus, the guidance of positioned points must be performed by controlling some points except the positioned points. This operation is referred to as *indirect simultaneous positioning* [2]. In this paper, we will focus on indirect simultaneous positioning as a fundamental operation of manipulation of deformable objects.

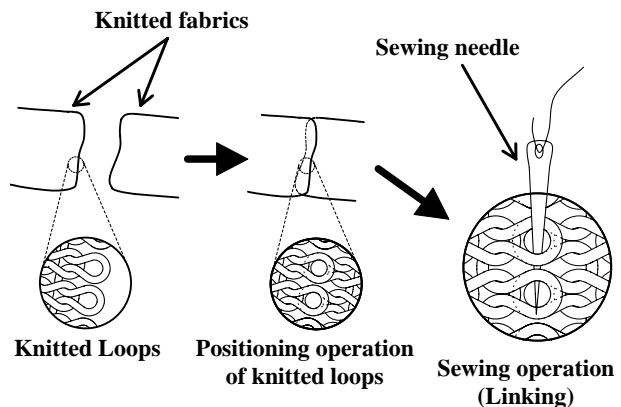


Figure 1: Linking of knitted fabrics

Some researches on manipulations of deformable objects have been conducted. For automated manufacturing of textile fabrics, many researches have been done [3]. Ono et al. [4] have derived a strategy for unfolding a fabric piece based on cooperative sensing of touch and vision. In these researches, since their approaches are for a specific task, thus it is difficult to apply the results to other different tasks in a systematic manner. Some researches have tried to deal with more general deformable object in systematic manners as follows. Hirai et al. [5] have proposed a method for modeling linear objects based on their potential energy and analyzed their static deformation. Wakamatsu et al. [6] have analyzed grasping of deformable objects and have introduced bounded force closure. In this approach, control of manipulative operations is out of consideration. Howard et al. [7] have proposed a method to model elastic objects by the connections of springs and dampers. A method to estimate the coefficients

of the springs and dampers has been developed by recursive learning method for grasping. This study has focused on model building. Thus, control problems for manipulative operations have not been investigated. Sun et al. [8] have studied on the positioning operation of deformable objects using two manipulators. They have focused on the control of the object position while deformation control is not discussed.

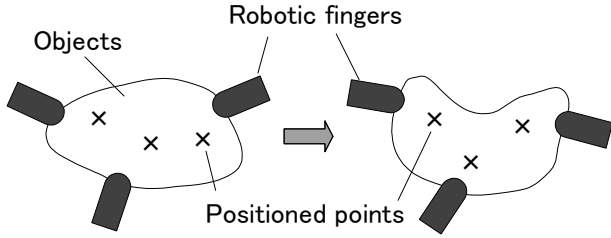


Figure 2: Indirect positioning of deformable object

In order to realize indirect simultaneous positioning, object model is important. However, it is difficult to build an exact model of the deformable objects in general due to nonlinear elasticity, friction, hysteresis, parameter variations, and other uncertainties. This is a main difficulty in manipulating deformable objects. To solve this dilemma, we have proposed to derive a robust manipulation strategy based on a coarse object model [2].

Based on this basic idea, we have proposed an iterative control scheme for the positioning [2]. It is, however, difficult to increase the speed of convergence in this approach. Also, it is difficult to maintain a stable grasp in grasping task as shown in Fig.2 because information of the deformation can only be utilized discretely [9]. Thus, it is desired to realize a new control law in which the deformation characteristics are utilized continuously.

Thus, we will realize the manipulation of deformable objects by a simple position feedback control that can utilize information of the deformation. In this control law, we only use a velocity relationship between positioned points and manipulation points. Then, the locations of the positioned points are measured in real-time, and the location is fed back to the robotic finger using the velocity relationship.

In this article, we will firstly build a coarse model of deformable objects. Next, indirect positioning will be formulated based on the coarse model. In the formulation, we will derive a relationship between velocity of the positioned points and that of manipulation points based on the equilibrium equations at the positioned points. Then, we will propose a feedback control law with an approximate model. Simulation results will show the validity and robustness of the proposed control law.

## 2 Formulation of Manipulation of Deformable Objects

### 2.1 Modeling of Deformable Objects

First of all, a model of deformable objects is derived. On modeling of deformable objects, many

researches have been conducted. For example in the area of computer graphics, cloth deformation is animated by Terzopoulos [10] or Louchet, Provot and Crochemore [11] and other many researchers. Our research goal is to realize robust manipulation of deformable objects. Therefore, we employ more simple deformation model. We model the object by connections of simple springs similar with Naster and Ayache [12]. For simplicity, we deal with two-dimensional deformable objects such as textile fabrics. We discretize the object by mesh points. Each mesh point is connected by vertical, horizontal, and diagonal springs as shown in Fig.3. In the model, we assume that the object deforms in a two-dimensional plane. In order to formulate the manipulation of deformable objects, object model must have the ability to describe translation, orientation, and deformation of the object simultaneously. Thus, position vector of the mesh points is utilized. Position vector of the  $(i, j)$ -th mesh point is defined as  $\mathbf{p}_{i,j} = [x_{i,j}, y_{i,j}]^T$  ( $i = 0, \dots, M; j = 0, \dots, N$ ). Coefficients  $k_x$ ,  $k_y$ ,  $k_\theta$  are spring constants of horizontal, vertical, and diagonal springs. Assume that no moment exert on each mesh point. Then, the resultant force exerted on mesh point  $\mathbf{p}_{i,j}$  can be described as eq.(1).

$$\mathbf{F}_{i,j} = \sum_{k=1}^8 \mathbf{F}_{i,j}^k = -\frac{\partial U}{\partial \mathbf{p}_{i,j}} \quad (1)$$

$U$  denotes whole potential energy of the object. Then, function  $U$  can be calculated by sum of all energies of springs [2]. Here, we assume that the shape of the object is dominated by eq.(1). Then, we can calculate the deformation of the object by solving eq.(1) under given constraints. Note that the following discussions are valid even if the object has an arbitrary three-dimensional shape by modeling the object similarly. Details have been reported in [2].

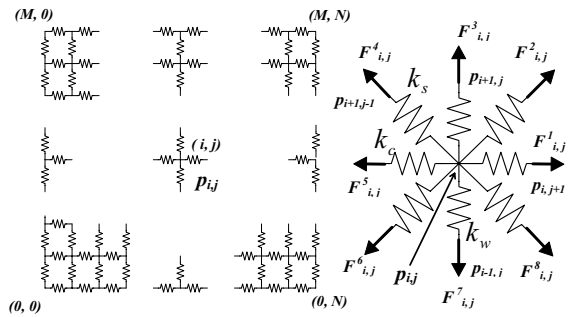


Figure 3: Spring model of deformable object

### 2.2 Problem Description

Here, we classify mesh points  $\mathbf{p}_{i,j}$  into the following three categories(see Fig.4) in order to formulate indirect simultaneous positioning.

**manipulation points:** are defined as the points that can be manipulated directly by robotic fingers. ( $\Delta$ )

**positioned points:** are defined as the points that should be positioned indirectly by controlling manipulation points appropriately. (○)

**non-target points:** are defined as the all points except the above two points. (others in Fig.4)

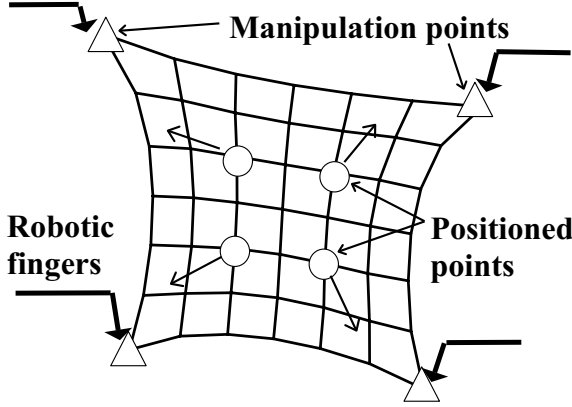


Figure 4: Classification of mesh point

Let the number of manipulation points and of positioned points be  $m$  and  $p$ , respectively. The number of non-target points is  $n = (M + 1) \times (N + 1) - m - p$ . Then,  $\mathbf{r}_m$  is defined as a vector that consists of coordinate values of the manipulation points. Vectors  $\mathbf{r}_p$  and  $\mathbf{r}_n$  are also defined for positioned and non-target points in the similar way. Eq.(1) can be rewritten as eqs.(2),(3) using  $\mathbf{r}_m$ ,  $\mathbf{r}_p$ , and  $\mathbf{r}_n$ .

$$\frac{\partial U(\mathbf{r}_m, \mathbf{r}_n, \mathbf{r}_p)}{\partial \mathbf{r}_m} - \boldsymbol{\lambda} = \mathbf{o}, \quad (2)$$

$$\begin{bmatrix} \frac{\partial U(\mathbf{r}_m, \mathbf{r}_n, \mathbf{r}_p)}{\partial \mathbf{r}_p} \\ \frac{\partial U(\mathbf{r}_m, \mathbf{r}_n, \mathbf{r}_p)}{\partial \mathbf{r}_n} \end{bmatrix} = \mathbf{o} \quad (3)$$

where a vector  $\boldsymbol{\lambda}$  denotes a set of forces exerted on the object at the manipulation points  $\mathbf{r}_m$  by robotic fingers.

Note that the external forces  $\boldsymbol{\lambda}$  can appear only in eq.(2), not in eq.(3). This implies that no external forces are exerted on positioned points and non-target points. These equations represent characteristics of indirect simultaneous positioning of deformable objects.

Let us consider the following task:

#### [Indirect Simultaneous Positioning (ISP)]

Assume that the configuration of robotic fingers and the positioned points on an object are given in advance. In addition, the robotic fingers pinch the object firmly. Then, the positioned points  $\mathbf{r}_p$  are guided to their desired location  $\mathbf{r}_p^d$  by controlling manipulation points  $\mathbf{r}_m$  appropriately.

### 3 Analysis of Instantaneous Motion of Deformable Object

#### 3.1 Velocity Relationship

Let us derive velocity relation among positioned points and manipulated points. We can obtain the following equation by differentiating eq.(3) by time with  $\mathbf{r}(t) = [\mathbf{r}_m^T(t), \mathbf{r}_n^T(t), \mathbf{r}_p^T(t)]^T$ :

$$A(\mathbf{r})\dot{\mathbf{r}}_m + B(\mathbf{r})\dot{\mathbf{r}}_n + C(\mathbf{r})\dot{\mathbf{r}}_p = \mathbf{o} \quad (4)$$

where

$$A(\mathbf{r}) \triangleq \begin{bmatrix} \frac{\partial^2 U(\mathbf{r})}{\partial \mathbf{r}_m \partial \mathbf{r}_p} \\ \frac{\partial^2 U(\mathbf{r})}{\partial \mathbf{r}_m \partial \mathbf{r}_n} \end{bmatrix} \in R^{(2p+2n) \times 2m},$$

$$B(\mathbf{r}) \triangleq \begin{bmatrix} \frac{\partial^2 U(\mathbf{r})}{\partial \mathbf{r}_n \partial \mathbf{r}_p} \\ \frac{\partial^2 U(\mathbf{r})}{\partial \mathbf{r}_n \partial \mathbf{r}_n} \end{bmatrix} \in R^{(2p+2n) \times 2n},$$

$$C(\mathbf{r}) \triangleq \begin{bmatrix} \frac{\partial^2 U(\mathbf{r})}{\partial \mathbf{r}_p \partial \mathbf{r}_p} \\ \frac{\partial^2 U(\mathbf{r})}{\partial \mathbf{r}_p \partial \mathbf{r}_n} \end{bmatrix} \in R^{(2p+2n) \times 2p}.$$

Vector  $\dot{\mathbf{r}}_m$  is defined as a velocity of the manipulation points. Vectors  $\dot{\mathbf{r}}_n$  and  $\dot{\mathbf{r}}_p$  are defined in the similar way. By transforming eq.(4), eq.(5) is obtained.

$$A(\mathbf{r})\dot{\mathbf{r}}_m + G(\mathbf{r})[\dot{\mathbf{r}}_n^T, \dot{\mathbf{r}}_p^T]^T = \mathbf{o}, \quad (5)$$

where

$$G(\mathbf{r}) \triangleq [B(\mathbf{r}) \ C(\mathbf{r})] \in R^{(n+p) \times (n+p)}. \quad (6)$$

Now, assume the follows:

**Assumption** The positions of positioned points  $\mathbf{r}_p$  can be determined uniquely, corresponding to the given positions of manipulation points  $\mathbf{r}_m$  for any number and configurations of the manipulation points.

From the Assumption, the infinitesimal displacement of the non-target points  $\delta \mathbf{r}_n$  and that of positioned points  $\delta \mathbf{r}_p$  can be determined uniquely corresponding to an arbitrary infinitesimal displacement of the manipulation points  $\delta \mathbf{r}_m$ . This yields the following [14].

Result 1

$$\text{rank}[B \ C] = 2p + 2n, \quad \forall m, \quad \forall \mathbf{r}_m, \quad \forall \delta \mathbf{r}_m$$

Then, velocity relation among manipulation points and positioned points can be derived as eq.(7) by transforming eq.(5).

$$\dot{\mathbf{r}}_p = J(\mathbf{r})\dot{\mathbf{r}}_m, \quad (7)$$

where  $J(\mathbf{r}) \triangleq -S_L G^{-1}(\mathbf{r})A(\mathbf{r}) \in R^{p \times m}$  and  $S_L \triangleq [0, I] \in R^{p \times (n+p)}$ . Note that the kinematic relationship among positioned points and manipulation points is included in matrix  $J(\mathbf{r})$  as well as the deformation model.

From the result 1, the next equation is satisfied.

$$S(B) \cap S(C) = \{0\}. \quad (8)$$

We obtain the following equation from eq.(4).

$$[A \ B] \begin{bmatrix} \delta r_m \\ \delta r_n \end{bmatrix} = -C\delta r_p \quad (9)$$

Here, we examine a condition that there exist displacements of operation points  $\delta r_m$  corresponding to an arbitrary  $\delta r_p$ . The necessary and sufficient condition that there exist displacements of the operation points  $\delta r_m$  corresponding to an arbitrary  $\delta r_p$  is given by

$$S([A \ B]) \supseteq S(C). \quad (10)$$

Considering (8) with eq.(10) yields

$$S(A) \supseteq S(C). \quad (11)$$

With Result 1, eq.(11) yields the following.

**Theorem 1** *There exist infinitesimal displacements of manipulated points  $\delta r_m$  corresponding to arbitrary infinitesimal displacements  $\delta r_p$ , if and only if,  $\text{rank}[A \ B] = 2p + 2n$  is satisfied.*

In addition, Theorem 1 needs the following result.

**Result 2** *The number of the manipulated must be greater than or equal to that of the positioned points in order to realize any arbitrary displacement  $\delta r_p$ , that is,  $m \geq p$ .*

Therefore, we assume  $m = p$ , the number robotic fingers equals to that of positioned points in this paper. Thus, Jacobian matrix  $J(\mathbf{r})$  also be a square one.

#### 4 Feedback Control Law

In this section, we propose simple PID feedback control laws in order to realize robust manipulation of deformable objects.

Assume that the position of the positioned points is measured by an image sensor and the velocity of the manipulation points is measured by an internal sensor of the robot fingers. In the manipulation of a deformable object, the velocity of the positioned points and that of manipulation points have to satisfy the relationship described by eq.(9). Thus, the following feedback control law is derived as similar as the control of robot manipulation in task space:

$$\mathbf{u} = -\hat{J}^T K_P(\mathbf{r}_p - \mathbf{r}_p^d) - K_V \dot{\mathbf{r}}_m - \hat{J}^T K_I \int (\mathbf{r}_p - \mathbf{r}_p^d) dt$$

where  $\mathbf{u}$  denotes the input torque or force to the joints of a robot manipulator and  $\hat{J}^T$  denotes the transpose of the Jacobian matrix  $J$  including some errors in deformation characteristics and the kinematic relation among points. In addition, we assume that a robot has prismatic joints when deriving eq.(12) for the sake of the simplicity.

Note that we can derive the similar control law even if the manipulator has rotational joints. Eq.(12) is similar with the robot control in task space with uncertain transposed Jacobian by Cheah et al [15].

#### 5 Simulation Results

In this section, simulation results are illustrated in order to show the validity and the robustness of the proposed control law represented by eq.(12).

The left side of Fig.5 shows the initial shape of the deformable object. Assume that the deforms in a 2-dimensional plane. the dimension of the object is  $60[mm] \times 60[mm]$ . There are four positioned points and four manipulation points as shown in Fig.5. Four crosses in the figure illustrates the desired locations of the positioned points. In this simulation, we define positioned points and manipulation points as follows:

$$\mathbf{r}_p = [\mathbf{p}_{1,1}^T, \mathbf{p}_{1,2}^T, \mathbf{p}_{2,1}^T, \mathbf{p}_{2,2}^T]^T \quad (13)$$

$$\mathbf{r}_m = [\mathbf{p}_{0,0}^T, \mathbf{p}_{0,3}^T, \mathbf{p}_{3,0}^T, \mathbf{p}_{3,3}^T]^T \quad (14)$$

The desired location of positioned points is given as follows:

$$\mathbf{r}_p^d = [100, 70, 120, 100, 80, 100, 100, 130]^T \quad (15)$$

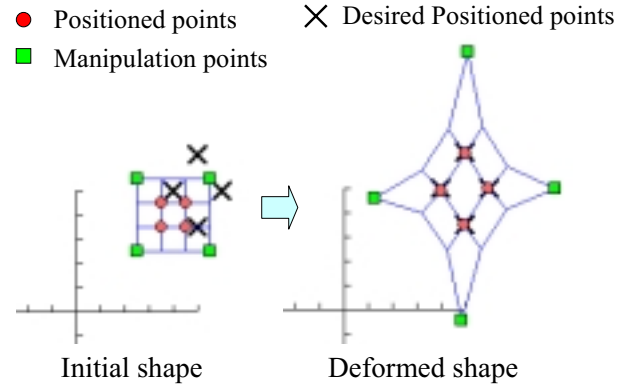


Figure 5: Simulation results

Recall that we have ignored masses and dampers of an object when we derived eqs.(12). But in the simulations, we suppose that the deformable objects have masses and dampers as well as springs. Namely, we employ the control law eq.(12) that is derived by ignoring mass and dampers while an object in simulations has masses and dampers. Thus, we also investigate whether the control law eq.(12) works effectively with the objects that has masses, dampers, and springs through the simulations. Assume that each lattice point has mass  $m = 0.01[\text{kg}]$  and each mesh has damping coefficient  $b = 5[\text{Ns/m}]$ . The spring constants is given by  $k_x = k_y = k_\theta = 10[\text{N/m}]$ . Each robotic finger has  $10[\text{kg}]$  mass. All feedback gains are fixed through the simulations.

Fig.6 illustrates the error norm of the simulation results. The solid line shows the results with exact object model. In the dotted line, we utilize  $k_x =$

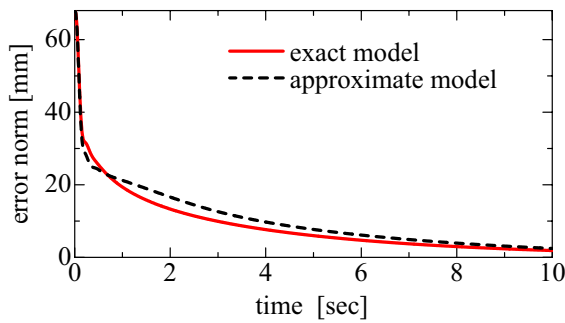


Figure 6: Error norm in simulation

$1[N/m]$ ,  $k_y = 10[N/m]$ ,  $k_\theta = 1[N/m]$  even though their exact values are  $k_x = k_y = k_\theta = 10[N/m]$ .

From Fig.6, we can conclude that the manipulation of the deformable objects can be realized by our proposed control law with approximate deformation model. The right hand side of Fig.5 illustrates the simulated shape with exact matrix. From this figure, our proposed control law realize large deformation. In addition, the proposed method can control the translation and the orientation of the object as well as the deformation as shown in Fig.5.

## 6 Conclusions

In this paper, manipulation of deformable objects has been formulated. We have proposed a position feedback control law with the approximate model. The validity and the robustness of the simple feedback has been shown through the simulation results.

The simulation results show that the error can converge even if the control law utilizes approximate model. The results shows that the method can control not only the deformation of the object but also the translation and the orientation simultaneously.

We will theoretically clarify how much deformations are acceptable when the approximate model is utilized as the future work.

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## References

- [1] Wada, T., Hirai, S., Hirano, T., Kawamura, S., "Modeling of Plain Knitted Fabrics for Their Deformation Control", Proc. of IEEE Int. Conf. on Robotics and Automation, pp.1960–1965, 1997
- [2] Wada, T., Hirai, S., Kawamura, S., "Planning and Control of Indirect Simultaneous Positioning Operation for Deformable Objects", Proc. of IEEE Int. Conf. on Robotics and Automation, pp.2572–2577, 1999
- [3] Taylor, P.M. et al.(Ed.), "Sensory Robotics for the Handling of Limp Materials", Springer-Verlag, 1990
- [4] Ono, E., Kita, N., Sakane, S., "Strategy for Unfolding a Fabric Piece by Cooperative Sensing of Touch and Vision", Proc. of Int. Conf. on Intelligent Robots and Systems, pp.441–445, 1995
- [5] Hirai, S., Wakamatsu, H., and Iwata, K., "Modeling of Deformable Thin Parts for Their Manipulation", Proc. IEEE Int. Conf. on Robotics and Automation, pp.2955–2960, 1994
- [6] Wakamatsu, H., Hirai, S., Iwata, K., "Static Analysis of Deformable Object Grasping Based on Bounded Force Closure", Proc. IEEE Int. Conf. on Robotics and Automation, pp.3324–3329, 1996
- [7] Howard, A.M. and Bekey, G.A., "Recursive Learning for Deformable Object Manipulation", Proc. of Int. Conf. on Advanced Robotics, pp.939–944, 1997.
- [8] Sun, D., Liu, Y., Mills, J.K., "Cooperative Control of a Two-Manipulator System Handling a General Flexible Object", Proc. of Int. Conf. on Intelligent Robots and Systems, pp.5–10, 1997
- [9] Wada, T., Hirai, S., Mori, H., Kawamura, S., "Robust Manipulation of Deformable Objects Using Model Based Technique", Proc. of Int. Workshop on Articulated Motion and Deformable Objects, in Lecture Notes in Springer Computer Science vol. 1899, pp.1–14, 2000
- [10] Terzopoulos, D., Platt, J., Barr, A., Fleischer, K., "Elastically Deformable Models", Proc. of Siggraph 87, Computer Graphics, 1987, Vol.21, No.4, pp.205–214
- [11] Louchet, J., Provot, X., Crochemore, D., "Evolutionary Identification of Cloth Animation Models", Computer Animation and Simulation'95: Proc. of the Eurographics Workshop in Maastricht, pp.44–54, 1995
- [12] Nastar, C. and Ayache, N., "Frequency-Based Nonrigid Motion Analysis: Application to Four Dimensional Medical Images", IEEE Trans. on Pattern Analysis and Machine Intelligence", Vol.18, No.11, pp.1067–1079, 1996
- [13] Wada, T., Hirai, S., Kawamura, S., "Indirect Simultaneous Positioning Operations of Extensionally Deformable Objects", Proc. of Int. Conf. on Intelligent Robots and Systems, pp.1333–1338, 1998
- [14] Wada, T., Hirai, S., Kawamura, S., "Analysis and Planning of Indirect Simultaneous Positioning Operation for Deformable Objects", Proc. of Int. Conf. on Advanced Robotics, pp.141–146 1999
- [15] Cheah, C.C., Kawamura, S., Arimoto, S., Lee, K., "PID Control of Robotic Manipulator with Uncertain Jacobian Matrix", Proc. ICRA'99, pp.494–499, 1999