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Task-selection Control by Thumb and Forefinger Based on Softfinger Contact

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Abstract: Human thumb and index finger are a superior combination for performing dextrous manipulation and secure pinch movements. Although many researchers who are active in the field of robotic hands have been concentrating on sophisticated control designs and their fully completed theoretical verifications, complicated computations containing system kinematics, dynamics, and Jacobian matrix based on those control designs are never implemented within the brain. This paper provides an extremely straightforward control scheme for achieving object orientation control on robotic pinch tasks, in which totally 1-DOF and 2-DOF robotic hands having the thumb and index finger manipulate the target object. Through the simulation of the pinch movements based on the proposed control design, it finally shows that non-Jacobian control can be accomplished by employing the characteristics of anatomical structure of the hand and placing a pair of soft fingertips at the distal ends of both the fingers.

Keywords: Softfinger, Thumb, Forefinger, Pinch motion, Manipulation, Jacobian matrix.

1. INTRODUCTION

Most of the mechanical structures of robotic manipulators are composed of only finite revolute joints through the whole mechanism. Currently, such manipulators have prevailed particularly in the research institute rather than the practical useage. This results in the fact that Jacobian matrix is able to describe the relationship between the task space and the joint space variables of the manipulators. Eventually, the description associated with the revolute joint angles for robots is more preferable rather than the prismatic joints. In addition, the Jacobian and its family (including transpose and inverse matrices) are usually used for computing joint torques required for force control strategy on the tip of the manipulator. As above, the Jacobian has been deeply into both the dynamics and kinematics of the robotic manipulators, since Dr. Carl Gustav Jacob Jacobi (1804-1851) has found out the functional matrix (Jacobian matrix) in his works. Now, can we eliminate the Jacobian family completely from robot control problems?

Generally, it had been usually said that human brain does not perform complicated computations about trajectory planning for their redundant arms or produce sophisticated control laws for dextrous manipulation or any other fine tasks. These observations are clearly verified because humans are able to accomplish multiple tasks very easily at a time. As a result, the traditional robot control scheme is far from an ultimate objective that robots get closer to the humans. Refined control designs that have been used to date for the robots and their performance should be just a supplementary tool for human-like robots. We presume that the physical movement function and anatomical bodily characteristics except higher-level brain functions extremely contribute to natural and high performances of the humans. It is obvious that relatively lower-level mechanical structure of the human body (e.g., musculotendon-skeletal mechanics) plays a key role in achieving even precise and delicate manipulation tasks through the tip of the index finger and the thumb. If the computation of the Jacobian family could be excluded from a series of controlling procedure execution for the manipulators and conventional robotic hands, it would be extremely groundbreaking to remove the large gap between the robots and humans.

This paper proposes a new control scheme that is able to complete precise orientation control of a target object grasped by an articulated multi-fingered robotic hand, on which a set of deformable soft fingertips are mounted in order to mimic the human fingertips and leverage their flexible mechanisms that will affect stable pinching motions. This control method is constructed on the basis of a sensory feedback design, which forms a *cascaded-twophased* structure in terms of the object orientation and joint angles of the hand. Particularly, the proposed control law has no Jacobian matrix in its closed-loop. In this case, we should notice that each joint is not necessary to be the revolute joint any more in the robotic system, because the Jacobian does not exist. It clearly means that a mixed mechanical structure including both the revolute and prismatic joints can be feasible for robot designs.

2. AN OBJECT ORIENTATION CONTROLLER DESIGN

2.1 Kinematic Thumb Models in Previous Studies

This study limits the operating range of the hand robot into a vertical two-dimensional plane (see Fig. 1) in which the influence of gravity is considered. From an anatomical viewpoint of finger movements, mechanical structure of the thumb has not been established yet. That is, the kinematic model of the thumb has not been uniquely standardized in biomechanical and robotic research field. For instance, Cooney et al. [1] postulated

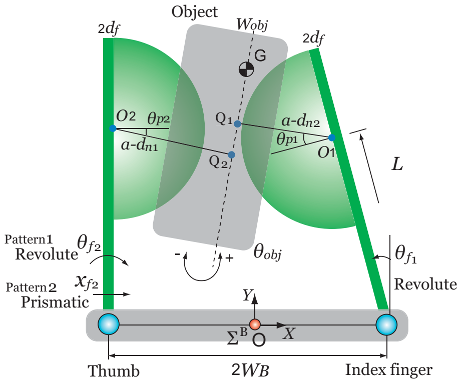


Fig. 1 Soft-fingered manipulation.

that during active motion such as pinch and grasp, muscle forces and the compressive force across adjacent joints constrain axial rotation of the finger and limit it to such an extent that the axial rotation is not considered as a degree-of-freedom of the thumb. As a result, a set of universal joints on both the metacarpophalangeal (MP) [2] and the trapeziometacarpal (TM) [2] joints of the thumb have been adopted, resulting in a five-DOFs kinematic model to describe the pinch motions. Giurintano et al. [3] also proposed a five-link kinematic model of the thumb, but which was different from the Cooney's model in the way that the universal joint structure was not employed for the reason that nonnegligible geometric offsets on each joint are present in terms of their extension/flexion and abduction/adduction movements. Note that the TM joint is often called carpometacarpal (CMC) joint in other studies [3–5]. On the other hand, in recent years it had been reported that the 5-DOF kinematic model was inherently not appropriate to describe the biomechanical thumb structure rigorously. In connection with the fact, Valero-Cuevas et al. [6] mentioned that the 5-DOF thumb model doubled the magnitude of the thumbtip forces and produced unrealistically large thumbtip torques in experimental pinch motions. In addition, Clewley et al. [7] postulated that the *effective number* of DOFs of every tasks is different one another, and knowing the number is also essential to mimic subtle and agile movements of human fingers in terms of musculoskeletal redundancy of the biomechanical finger structure. As above, nothing of generally-acceptable full-link mechanisms of the thumb based on the anatomical knowledge had been formulated so far.

In this paper, as shown in Fig. 1 we consider an extremely simple robotic hand structure (2-DOF) that consists of an opposed pair of an index finger and a thumb, on which a set of deformable soft fingertips are mounted for stable manipulation. This opposed mechanism was assumed for such a reason that we have wondered if expected feasible tasks using the individual mechanism of revolute and prismatic joints differ from each other during pinch motions.

In fact, the translational movement of the distal link is readily confirmable from human thumb motion shown in Fig. 2. This can also be theoretically indicated using a 3-DOF manipulator mechanism shown in Fig. 3, which is required as a minimal number of DOFs for achieving the translational movement at the distal link. Thus, as a first step of our research, this paper confines the model to a simple 3-DOF revolute joint mechanism for the thumb in the two-dimensional plane. In particular, this study concentrates on a minimal-DOF configuration of the opposed formation, which can be composed by treating a pair of distal links located at each of the index finger and the thumb, in order to properly evaluate the relationship between the joint structure and the feasible tasks and to exclude the influence of mechanical redundancy and its associated control issues.

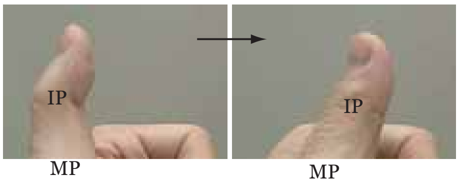


Fig. 2 Prismatic movement of the distal phalange of the thumb.

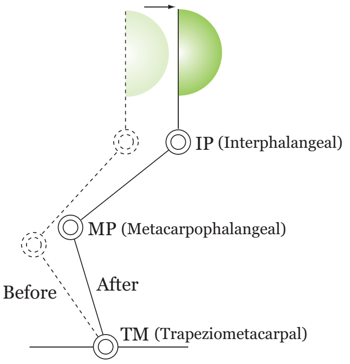


Fig. 3 A 3-DOF finger mechanism in the two-dimensional plane and the prismatic movement of the distal link.

2.2 Equations of Motion

Equations of motion of the two-fingered manipulation expressed as a dynamic constrained system which is having a pair of revolute joints (RR mechanism in short) have already been represented in our previous study [8]. Hence, in this section we formulate a set of dynamic equations of motion of a pair of revolute and prismatic joints (RP mechanism in short) on the basis of Lagrange's method in the presence of dynamically-changeable constraints during manipulation tasks.

As shown in Fig. 1, let $G(x_{obj}, y_{obj})$ be the center of gravity of a grasped object, θ_{obj} be the orientation angle of the object with respect to the base coordinate system $\sum^B$. In addition, let W_{obj} be an object width, $2W_B$ be the distance between both joints of the hand, L be the length of both fingers, $2d_f$ be the width of each finger, and O_i be the origin of i-th fingertip. This study classifies a couple of patterns of joint motions individually (see Fig. 1) and attempts to discuss the different manipu-

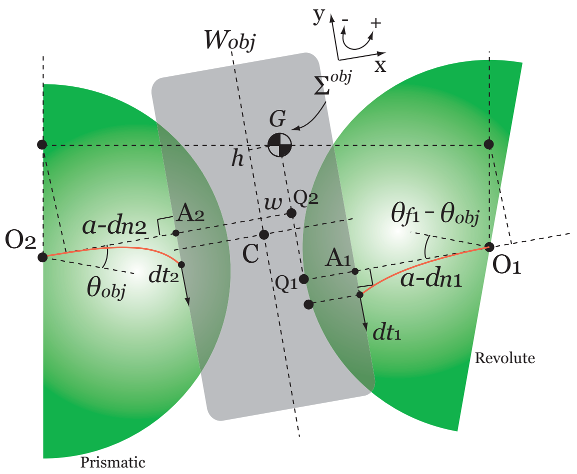


Fig. 4 The geometric relationship on the soft-fingered grasping is illustrated.

lation capability of each of the RR and RP joint mechanisms. Therefore, a set of system variables for the robotic hand system are defined as $(x_{obj}, y_{obj}, \theta_{obj}, \theta_{f1}, \theta_{f2})$ or $(x_{obj}, y_{obj}, \theta_{obj}, \theta_{f1}, x_{f2})$, respectively. Particularly, in the case of the latter structure of the hand, equations of motion of the total system are described in what follows.

The origin of both the fingertips is represented as

$$\begin{aligned} O_{1x} &= W_B - L\sin\theta_{f1} - d_f\cos\theta_{f1}, \\ O_{1y} &= L\cos\theta_{f1} - d_f\sin\theta_{f1}, \\ O_{2x} &= -W_B + d_f + x_{f2}, \\ O_{2y} &= L. \end{aligned} \tag{1}$$

Recalling the geometric constraint that appears along to the normal direction of the grasped object [8], it is rewritten as

$$\begin{aligned} C_{ni} = & (-1)^i(x_{obj} - O_{ix})\mathrm{C_{obj}} \\ & +(-1)^i(y_{obj} - O_{iy})\mathrm{S_{obj}} \\ & -(a - d_{ni}) - \frac{W_{obj}}{2} = 0, \end{aligned} \tag{2}$$

where the simplified form, $\mathrm{S_{obj}}$ and $\mathrm{C_{obj}}$, denote $\sin\theta_{obj}$ and $\cos\theta_{obj}$, respectively. In Eq. (2), the first and second terms on the right side mean the distance between the center of gravity of the object and point O_i, which remains normal to the object contacting surface.

On the other hand, *pfaffian constraints* [9, 10] governing tangential motions of the object and soft fingertip can be expressed by considering velocities of both displacements due to object rolling motions. Note that in this study effective rolling radii on the both fingertips are assumed to be a that is equivalent to the mechanical radius of the fingertips on the basis of Chang's study [11]. As shown in Fig. 4, let GQ_i be an intersection between the straight line O_iA_i and a line extended from the point G parallel to the y-axis of the object coordinate system $\sum^{obj}$. The distance GQ_i is then represented as

$$\begin{aligned} GQ_i = & -(x_{obj} - O_{ix})\sin\theta_{obj} \\ & +(y_{obj} - O_{iy})\cos\theta_{obj}. \end{aligned} \tag{3}$$

When the grasped object rotates towards positive direction (counterclockwise) on the right fingertip with maintaining the effective rolling radius a, then GQ_1 increases gradually. However, even if the object simultaneously slides along to the y-direction with a certain deflection d_{t1} of the fingertip (see Fig. 4), point Q_1 itself does not change from the extended line of O_1A_1 as long as the definition of Q_1 is preserved, resulting in shortening GQ_1. As a result, GQ_1 lengthens due to the positive rolling motion but shortens due to the slide of the object. Hence, this geometrically-combined relationship can be expressed using velocity form description as

$$\dot{C}_{ti} = \dot{GQ}_i + a\dot{\theta}_{pi} + \dot{d}_{ti} = 0, \tag{4}$$

where

$$\theta_{p1} = \theta_{f1} - \theta_{obj}, \quad \theta_{p2} = \theta_{obj}. \tag{5}$$

Note that Eq. (4) corresponds to a set of pfaffian constraints in a minimal-DOF soft-fingered manipulation.

Generally, Lagrange equations of motion of dynamically-constrained systems are indicated as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} - \boldsymbol{f}_p - \boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\lambda} = \boldsymbol{f}_{ext} + \boldsymbol{u}, \tag{6}$$

where $\boldsymbol{M}$ is an inertia matrix containing rotational and translational movements of the present system, and $\boldsymbol{D}$ denotes a viscous damping matrix that is the intrinsic parameter of soft finger materials. In addition, $\boldsymbol{q}$ stands for a set of system variables $(x_{obj}, y_{obj}, \theta_{obj}, \theta_{f1}, x_{f2})$ in the case of RP joint mechanism, and $\boldsymbol{f}_p$ is a vector of generalized potential forces/moments, which is expressed by differentiating each of elastic potential energy, P_e, due to deformation of the fingertips and gravitational potential energy, P_{gv}, with respect to the generalized coordinate, and given as

$$\boldsymbol{f}_p = -\left\{\frac{\partial P_e}{\partial \boldsymbol{q}} + \frac{\partial P_{gv}}{\partial \boldsymbol{q}}\right\}. \tag{7}$$

$\boldsymbol{\Phi}$ is a constraint matrix [12] obtained by differentiating each constraint equation with respect to $\boldsymbol{q}$ or $\dot{\boldsymbol{q}}$. In the present system, it can be described by the following calculations:

$$\boldsymbol{\Phi} = \left[\left(\frac{\partial \boldsymbol{C}_n}{\partial \boldsymbol{q}}\right)^{\mathrm{T}}, \left(\frac{\partial \dot{\boldsymbol{C}}_t}{\partial \dot{\boldsymbol{q}}}\right)^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{8}$$

$\boldsymbol{\lambda}$ means undetermined multipliers that further correspond to constraint forces caused by the mechanical contact between the grasped object and both fingertips. $\boldsymbol{f}_{ext}$ denotes external forces and moments applied to the system and $\boldsymbol{u}$ indicates control input signals for the robotic hand.

2.3 A Cascaded-twophased Controller

Recall a fundamental cascaded-twophased controller capable of achieving robust convergence of the orientation of grasped object as follows:

$$\theta_{fi}^d = -(-1)^i K_I^\theta \int_0^t (\theta_{obj} - \theta_{obj}^d)\mathrm{d}\tau, \tag{9}$$

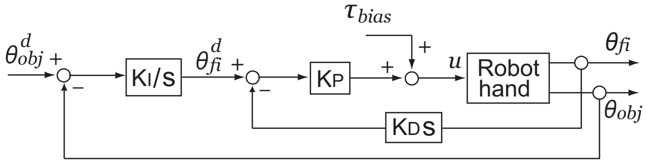


Fig. 5 A block diagram of cascaded-twophased controller.

Table 1 Simulation parameters

parameters	values (orientation control)
θ^d_{obj}	$-2° \to -5° \to -8°$
K^θ_P	300 Nm
K^θ_D	14 Nm·sec
K^θ_I	0.003
τ_{bias}	10 Nm
	values (position control)
x^d_{obj}	4 mm → −6 mm → −6 mm
K^x_P	300 N/m
K^x_D	14 N·sec/m
K^x_I	0.1
τ_{bias}	10 N

$$u_i = -K^\theta_P(\theta_{fi} - \theta^d_{fi}) - K^\theta_D \dot{\theta}_{fi} + \tau_{bias}, \quad (10)$$

where K^θ_P, K^θ_D, and K^θ_I respectively denote the proportional, derivative, and integral gains for the controller in which the superscript θ means that these gains are specially used for the object orientation control. This controller architecture adopted a discriminative design in appearance such that desired joint angles are dynamically and continuously produced during the manipulation control and be serially-cascaded between the first and second stages represented as Eqs. (9) and (10), as shown in Fig. 5. Note that this controller has no Jacobian matrix, especially, the biased torque τ_{bias} in the second phase is an arbitrary decidable constant, but not be the one obtained by multiplying the transpose of Jacobian matrix into certain grasping forces. This method can be employed in the case of the revolute joint mechanism illustrated as the pattern 1 in Fig. 1. By utilizing the proposed controller and evaluating its control performances, we verify using numerical simulations that non-Jacobian control can be achieved in soft-fingered robotic manipulation.

3. SIMULATION STUDY

3.1 A Revolute Joint vs. A Prismatic Joint (RP joints)

To begin with, introducing an another set of cascaded-twophased controller designed for the object position control (see Fig. 1), it can be given by modifying the previous orientation controller as

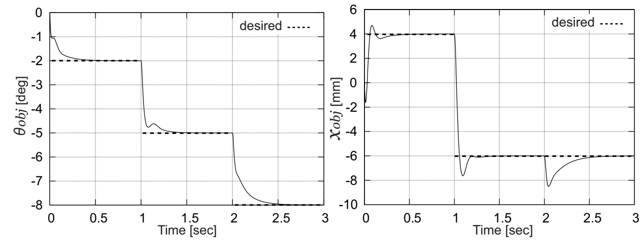
$$x^d_{f2} = -K^x_I \int_0^t (x_{obj} - x^d_{obj})\mathrm{d}\tau, \quad (11)$$

$$u_2 = -K^x_P(x_{f2} - x^d_{f2}) - K^x_D \dot{x}_{f2} + \tau_{bias}. \quad (12)$$

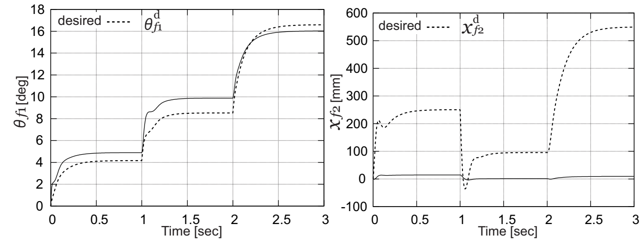
In the above equations, K^x_P, K^x_D, and K^x_I denote the proportional, derivative, and integral gains respectively.

Table 2 Mechanical parameters

parameters	values[mm]
L	76.2
a	20
d_f	4
$2W_B$	98
W_{obj}	50



(a) θ_{obj} and θ^d_{obj} (b) x_{obj} and x^d_{obj}



(c) θ_{f1} (d) x_{f2}

Fig. 6 Simulation results of independent-simultaneous control of the position and orientation of the RP mechanism).

Taken together, Eqs. (11) and (12) are utilized to control the object position and the prismatic joint, and Eqs. (9) and (10) to which $i = 1$ is substituted are employed for the object orientation and the rotational index finger. All parameters used in both the controllers are listed in Table 1. A straightforward task given to the robotic hand in this simulation is simultaneous control of the object position and orientation, whose desired values $(\theta^d_{obj}, x^d_{obj})$ are also listed in Table 1. The designated pattern is set to vary at the interval of 1 sec. Finally, we can examine the dynamic behavior of the total robotic hand system by carrying out numerical computation for Eq. (6) together with the control inputs (u_1, u_2).

The initial setup of the robot configuration related to the base coordinate $\sum^B$ is assumed such that (see Fig. 1)

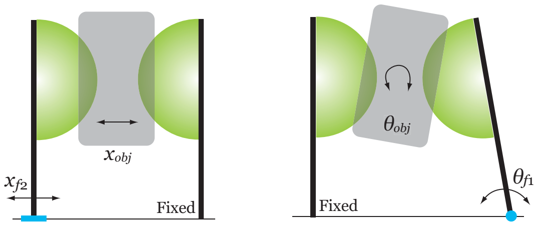
$$\begin{aligned} x_{obj} &= 0\ \text{mm}, & \theta_{obj} &= 0°, \\ \theta_{f1} &= 0°, & x_{f2} &= -49\ \text{mm}. \end{aligned} \quad (13)$$

This means that both the fingers grasp a planar object maintaining geometric parallel between the object and both the fingers in the initial configuration. From the initial condition, a set of desired trajectories $(x^d_{obj}, \theta^d_{obj})$ are given to the system equations at a time. Fig. 6 shows simulation results in the RP joint mechanism. As shown in Fig. 6-(a) and (b), x_{obj} and θ_{obj} both converge to the desired trajectory precisely. We can further find that θ_{obj} is able to vary with no change of the x_{obj} trajectory. This fact means that the position and orientation controls can be separated and be controlled independently, resulting in an observation that the revolute joint contributes only to the orientation control of grasped object, and on the other hand, the prismatic joint acts as functions only for controlling the object position. That is, there exists a certain kind of role-sharing for pinch tasks between the index finger and thumb when employing different types of joints (RP joints) with which adjoining target object is in contact. It is also obvious that these new findings can be explained by a concept of LMEE (Local Minimum of Elastic Potential Energy) of soft fingertips, which was demonstrated in our previous study [13].

In addition, Fig. 6-(c) and (d) show that the revolute and prismatic joints both converge to a certain angle and position, respectively. However, there exist clearly large errors in every time steps. Particularly, the position trajectory of the prismatic joint is completely out of control throughout the simulation in the sense that the discrepancy becomes extremely large. It is therefore likely to consider that the control design method proposed in this paper leads to a failure apparently. These discrepancies, however, would become obviously permissible errors as long as the object position and orientation have converged to each desired value. In fact, the cause of this phenomenon is intimately linked with the LMEE.

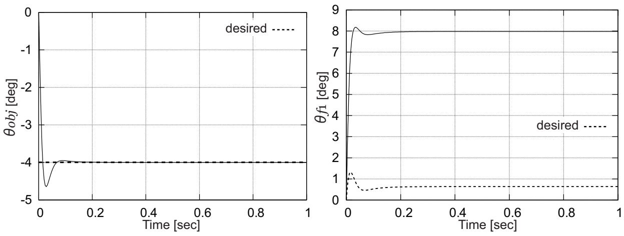
The LMEE, to begin with, has been found on the basis of a physically spontaneous idea that elastic deformations of soft fingertips have some sort of minimum level on their elastic energy [13]. Particularly in case of actual pinch motions, since any multiple geometric and kinematic constraints absolutely exist during dynamic manipulation tasks, this extended perspective of elastic energy has been defined as *LMEE with constraints* (LMEEwC in short) [8]. The LMEEwC corresponds to an equilibrium point in terms of physical meanings, therefore in this case, a set of system variables that ought to be the equilibrium point are uniquely determined as, e.g. $(x^{\star}_{obj}, y^{\star}_{obj}, \theta^{\star}_{obj}, \theta^{\star}_{f1}, x^{\star}_{f2})$. In order to simultaneously control two or more system variables among the set of the unique solution, i.e. LMEEwC, the chosen variables must be preliminarily known to avoid such inconsistency that either variable is not the member of the LMEEwC. This equilibrium point, however, cannot be analytically computed due to the strong nonlinearity of the deformation model of the elastic soft fingertips.

Now, in the present study, when a desired object position x^{d}_{obj} decided arbitrarily is assumed to be an element among the LMEEwC, the desired joint displacement x^{d}_{f2} of Eq. (11), which is artificially produced and depends on the integral gain K^{x}_{I}, it never becomes the element of the LMEEwC unlike the x^{d}_{obj}. The joint displacement x_{f2} indicated in Eq. (12), therefore, must not converge to x^{d}_{f2} with no error. If it does, the convergence will become a cause to disturb the physical stabilization of the LMEEwC. The PD control for the joints expressed in Eqs. (10) and (12) is able to remain errors constitutionally, which as a matter of fact correspond to a certain kind of *allowable escape* associated with the joint displacement and joint angle to discourage from the lack of the stability. As a result, the existing discrepancies depicted in Fig. 6-(c) and (d) must be preserved due to the above reasons. We can thereby conclude that the actually converged values (continuous lines) seen in every time steps coincide to each member of the LMEEwC.



(a) translational movement (b) rotational movement

Fig. 7 A simple conceptual diagram of 1-DOF hand.

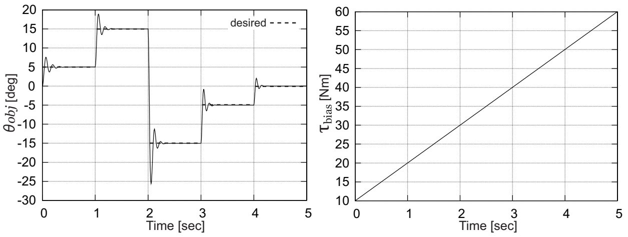


(a) θ_{obj} (b) θ_{f1}

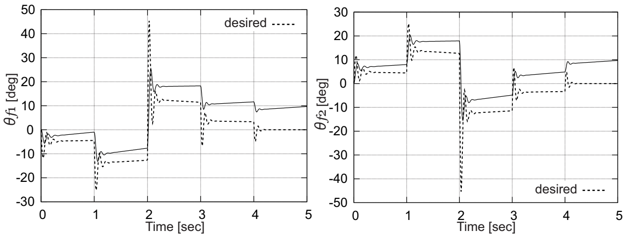
Fig. 8 Simulation results of the object orientation control on the mechanical structure of Fig. 7-(b).

From the viewpoint of the LMEEwC, another problem still remains unsolved why both the desired values of x^{d}_{obj} and θ^{d}_{obj} could be independently decided. To elucidate the problem, we consider extremely simple robotic hands having only single 1-DOF prismatic or revolute joint, that is, we simulate two cases that either the thumb or the index finger is fixed (see Fig. 7). In that case, what sort of movements and tasks can each remaining finger perform?

It is clear in case of a pair of the prismatic and a fixed joint that the position control of grasped object along to the translational direction equivalent to the joint motion can be carried out (see Fig. 7-(a)). On the other hand, in the other case of the revolute joint (see Fig. 7-(b)), Fig. 8 shows that the orientation control of the object can be realized despite the considerably large deviation of the revolute joint. These overall results can be interpreted as that the index finger of revolute joint assumes an orientation control and the thumb of prismatic joint fulfills a role for position control of the grasped object. These new findings accomplish an individual control of robotic joints configured with a pair of different mechanisms. In other words, the functional role-sharing of the index finger and thumb for complicate pinch motions simplifies even the traditional control architecture of multi-fingered robotic hands. This implies that a control scheme without the Jacobian matrix family is a feasible methodology as long as the deformable soft fingers mounted on the distal links of the hand grasp and manipulate the target object. We can thus conclude that the LMEEwC is able to powerfully ensures stable and robust pinch motions and relates directly to the possibility of non-Jacobian control newly proposed in this study.



(a) θ_{obj} and θ^d_{obj} (b) increment of τ_{bias}



(c) θ_{f1} (d) θ_{f2}

Fig. 9 Simulation results of the object orientation control with continuous increase of the biased torque.

3.2 A Revolute Joint vs. A Revolute Joint (RR joints)

We can expect from the previous results and above observations that the object orientation can at least be controlled in the case of RR joint mechanism. This prediction can easily be seen in Fig. 9-(a). This shows that the object orientation robustly converges to each desired value that varies every second.

Now, reviewing the results of RP joints, we know that the position and orientation of the object could be controlled precisely, indicating that the double outputs (x_{obj}, θ_{obj}) are regulated by the double inputs (x_{f2}, θ_{f1}), respectively. Then, in the case of RR joints one question may arise: what is another achievable remaining task (output) other than the object orientation in terms of the double inputs $(\theta_{f1}, \theta_{f2})$. Fig. 9 gives an answer that is the biased torque τ_{bias} involved in Eq. (10). The torque is a positive constant that can be arbitrarily adjusted, resulting in secure grasping and an effect such that the input torque u_i (Eq. (10)) totally remains positive. The increase of the torque affects that of grasping forces without any movement of both finger joints in the conventional approach for robotic hand applications. The case of soft fingers, however, differs in the sense that both the fingers produce inward rotation due to the further fingertip deformation which occurs along with the increased torque. Fig. 9-(b) shows a given pattern of the linear increase of τ_{bias}. Despite the continuous increase, the object orientation precisely converges and the allowable errors of the RR joints clearly appear as shown in Fig. 9-(c) and (d).

4. CONCLUDING REMARKS

This study has demonstrated that the biomimicry of the anatomical hand structure has a potential to drastically simplify the control architecture of robotic hands. The common knowledge of manipulator control such a Jacobian matrix had been interfering with the findings of hidden capabilities of complex mechanical structure including the revolute and prismatic joints. The distinctive biomechanical configuration ingenerates a certain role-sharing function that contains both the different kinds of mechanical joints. This study has finally provided some perspectives obtained on the basis of our simulational observations, and given as follows:

(1) The prismatic joint correlates only to the object position control.

(2) The revolute joint correlates only to the object orientation control.

(3) The different pair of joints enable to independent-simultaneous control of both the position and orientation of the object.

(4) The same pair of joints further enable to control the grasping torque in addition to the corresponding position or orientation of the object.

REFERENCES

[1] W.P.Cooney, M.J.Lucca, E.Y.S.Chao, and R.L.Linscheid, "The Kinesiology of the Thumb Trapeziometacarpal Joint", *Journal of Bone and Joint Surgery*, Vol.63, No.9, pp.1371-1381, 1981.

[2] A.I.Kapandji, "The Physiology of the Joints, Volume 1: Upper Limb", *Churchill Livingstone*, 6th edition, 2007.

[3] D.J.Giurintano, A.M.Hollister, W.L.Bufold, D.E.Thompson, and L.M.Myers, "A virtual five-link model of the thumb", *Medical Engineering and Physics*, Vol.17, No.4, pp.297-303, 1995.

[4] L.Y.Chang and Y.Matsuoka, "A Kinematic Thumb Model for the ACT Hand", *Proc. of IEEE Int. Conf. Robotics and Automation*, pp.1000-1005, 2006.

[5] A.Hollister, W.L.Bufold, L.M.Myers, D.J.Giurintano, and A.Novick "The Axes of Rotation of the Thumb Carpometacarpal Joint", *Journal of Orthopaedic Research*, Vol.10, No.3, pp.454-460, 1992.

[6] F.J.Valero-Cuevas, M.E.Johanson, and J.D.Towles, "Towards a realistic biomechanical model of the thumb: the choice of kinematic description may be more critical than the solution method or the variability/uncertainly of musculoskeletal parameters", *Journal of Biomechanics*, Vol.36, pp.1019-1030, 2003.

[7] R.H.Clewley, J.M.Guckenheimer, and F.J.Valero-Cuevas, "Estimating Effective Degrees of Freedom in Motor Systems", *IEEE Trans. Biomedical Engineering*, Vol.55, No.2, pp.430-442, 2008.

[8] T.Inoue and S.Hirai, "A Two-phased Object Orientation Controller on Soft Finger Operations", *Proc. of IEEE Int. Conf. Intelligent Robots and Systems*, pp.2528-2533, 2007.

[9] R.Murray, Z.Li, S.Sastry, "A Mathematical Introduction to Robotic Manipulation", CRC Press, 1994.

[10] M.Spong, S.Hutchinson, and M.Vidyasagar, "Robot Modeling and Control", John Wiley & Sons Inc, 2005.

[11] D.Chang and M.Cutkosky, "Rolling with Deformable Fingertips", *Proc. IEEE/RSJ Int. Conf. Intelligent Robots and Systems*, pp.194-199, 1995.

[12] T.Park and E.Haug, "A Hybrid Numerical Integration Method for Machine Dynamic Simulation", *ASME Trans. Journal of Mechanisms, Transmissions, and Automation in Design*, Vol.108, pp.211-216, 1986.

[13] T.Inoue and S.Hirai, "Elastic Model of Deformable Fingertip for Soft-Fingered Manipulation", *IEEE Trans. Robotics*, Vol.22, No.6, pp.1273-1279, 2006.