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## DEFORMATION PATH PLANNING FOR BELT OBJECT MANIPULATION

Yuya ASANO, Hidefumi WAKAMATSU, Eiji MORINAGA, Eiji ARAI  
Graduate School of Engineering Osaka University  
2-1 Yamadaoka, Suita, Osaka 565-0871, Japan  
y-asano@mapse.eng.osaka-u.ac.jp

Shinichi HIRAI  
Ritsumeikan University  
1-1-1 Noji Higashi Kusatsu Shiga 525-8577, Japan

### ABSTRACT

A differential geometry based modeling to represent deformation path for a belt object is proposed. Adequate deformation path of a belt object such as film circuit board or flexible circuit board must be generated for automatic manipulation and assembly. First, deformation of a belt object is described using the direction of the central axis and torsion around it. Second, a method to derive an adequate transition of the object shape from the initial state to the final state is proposed. The adequate deformation transition can be derived by minimizing the maximum potential energy during its manipulation process. Flexible circuit board can be easily bent along the central axis, but it must not be twisted around the central axis because it may cause the crack at the transverse edges leading to wiring disconnection. So, more suitable deformation path is considered so that the stress in a belt object is small.

### INTRODUCTION

Due to downsizing of various electronic devices such as notebook PCs, mobile phones, digital cameras, and so on, more film circuit boards or flexible circuit boards illustrated in Fig.1 are used instead of conventional hard circuit board. It is difficult to assemble such flexible boards by a robot because they can be easily deformed during their manipulation process and they must be deformed in the final state. For example, the flexible circuit board shown in Fig.1-(a) must deform to the objective shape illustrated in Fig.1-(b) to install into the hinge part of a flip phone. Therefore, analysis and estimation of film/flexible circuit boards is required.

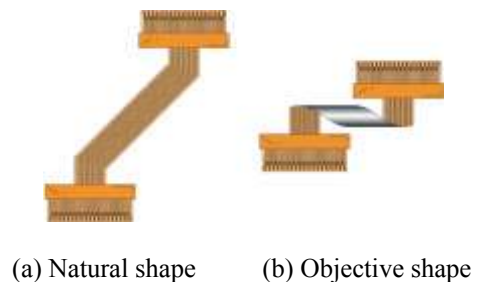


Fig. 1. Example of flexible circuit board

In computer graphics, a deformable object is represented by a set of particles connected by mechanical elements[1]. Recently, fast algorithms have been introduced to describe liner object deformation using Cosserat formulation[2]. Cosserat elements possess six degrees of freedom; three for translation displacement and three for rotational displacement. Flexure, torsion, and extension of a liner object can be described by use of Cosserat elements. In robotics, insertion of a wire into a hole in 2D space has been analyzed using a beam model of the wire to derive a strategy to perform the insertion successfully[3][4]. Kosuge et al. have proposed a control algorithm of dual manipulators handling flexible sheet metal[5]. Lamiraux et al. have proposed a method of path planning for elastic object manipulation with its deformation to avoid contact with obstacles in a static environment[6]. Saha and Isto proposed a motion planner for manipulating ropes and realized tying several knots using two cooperating robot arms[7]. In differential geometry, curved lines in 2D or 3D space have been studied to describe their shapes mathematically[8]. Moll et al.

have proposed a method to compute the stable shape of a liner object under some geometrical constraints quickly based on differential geometry[9]. It can be applied to path planning for flexible wires. Jian proposed the modeling method for deformable shell-like object[10].

As above, the modeling method to represent deformable object deformation and the method to manipulate it has been proposed. However, to my knowledge, there is not a research to estimate the deformed shape and load condition of it during its manipulation process. A flexible circuit board is a deformable object. So, it is possible to be deformed to contact with itself or obstacles in a static environment. In addition, a flexible circuit board is precision mechanical equipment. This means that it is possible to make fracture when it is deformed largely during its manipulation process. It can be easily bent along the central axis, but it must not be twisted around the central axis because it may cause the crack at the transverse edges leading to wiring disconnection. So, it is required to estimate the deformation shape and load condition of the object during its manipulation process.

In solid mechanics, the Kirchhoff theory for thin plates and Reissner-Mindlin theory for thick plates have been used[11]. For extremely thin plates, the inextensional theory was proposed[12]. In this theory, it is assumed that the middle surface of a plate is inextensional, that is, the surface of the plate is developable. Based on these theories, the deformed shape and load condition of the object can be calculated using the Finite Element Method (FEM). However, the high aspect ratio of thin objects often causes instability in computation of deformed shapes. Wakamatsu has proposed a differential geometry to represent linear object deformations and path planning[13]. In this paper, we apply this theory to deformation and deformation path of a belt object and propose the manipulation planning so that the object deforms with little damage during its manipulation process.

## MODELING OF BELT OBJECT

### DESCRIPTION OF DEFORMED SURFACE

In the case of a thin object, one dimension is assumed to be successfully small comparing with the other two dimensions, namely,  $d_1 \ll d_2, d_3$ . In this paper, we define an object with the following aspect ratio as a *belt object*:  $d_1 \ll d_2 \ll d_3$ . This implies that the thickness  $h$  of the object is sufficiently smaller than its width  $b$ , and its width  $b$  is sufficiently smaller than its length  $L$ . In manipulation of belt objects such as film/flexible circuit boards, flat cables, and so on, they bend and twist mainly and their expansion/contraction can be negligible. In differential geometry, a 3D surface which can be flattened without its expansion or contraction is defined as a *developable surface*. So, the deformed shape of an inextensible belt object corresponds to a developable surface.

Let  $u$  be the distance from one end of the object along the central axis in its longitudinal direction and let  $v$  be the distance from the central axis in the transverse direction of the object. Let  $P(u, v)$  be a point on the object. In order to describe deformation of the central axis of a belt object, the global space

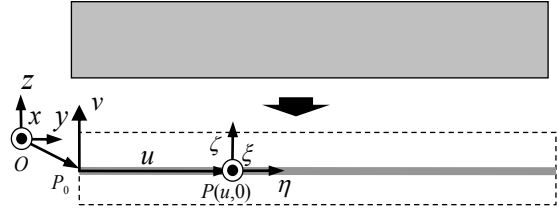


Figure2: Coordinate of belt object

coordinate system and the local object coordinate systems at individual points on the object are introduced as shown in Fig.2. Let  $O-xyz$  be the coordinate system fixed in space and  $P-\xi\eta\zeta$  be the coordinate system fixed at an arbitrary point  $P(u, 0)$  on the central axis of the object. Select the direction of coordinates so that the  $\xi$ -,  $\eta$ -, and  $\zeta$ -axes are parallel to the  $x$ -,  $y$ -, and  $z$ -axes, respectively, in the natural state. Deformation of the object is then represented by the relationship between the local coordinate system  $P-\xi\eta\zeta$  at each point on the object and the global coordinate system  $O-xyz$ . Let  $\xi$ ,  $\eta$ , and  $\zeta$  be unit vector along the  $\xi$ -,  $\eta$ -, and  $\zeta$ -axes, respectively, at any point  $P(u, 0)$ . By analogy with angular velocities of a rigid object, partial differentiation of these unit vector with respect to  $u$  are described by

$$\frac{d}{du} \begin{bmatrix} \xi & \eta & \zeta \end{bmatrix} = \begin{bmatrix} \xi & \eta & \zeta \end{bmatrix} \begin{bmatrix} 0 & -\omega_\zeta & \omega_\eta \\ \omega_\zeta & 0 & -\omega_\xi \\ -\omega_\eta & \omega_\xi & 0 \end{bmatrix} \quad (1)$$

where  $\omega_\xi$ ,  $\omega_\eta$ , and  $\omega_\zeta$  are infinitesimal ratios of rotation angles around the  $\xi$ -,  $\eta$ -, and  $\zeta$ -axes, respectively, at point  $P(u, 0)$ . Note that  $\omega_\xi$  and  $\omega_\zeta$  correspond to curvatures of central axis in the  $\eta\zeta$ -plane and in the  $\xi\eta$ -plane respectively, and  $\omega_\eta$  correspond to torsional ratio around the central axis. Solving differential equations described by eq.(1) numerically, vector  $\xi$ ,  $\eta$ , and  $\zeta$  at point  $P(u, 0)$  can be determined. Let  $\mathbf{x}(u, 0) = [x(u, 0) \ y(u, 0) \ z(u, 0)]^T$  be the position vector at  $P(u, 0)$ . The position vector can be computed by integrating vector  $\boldsymbol{\eta}(u, 0)$ . Namely,

$$\mathbf{x}(u, 0) = \mathbf{x}_0 + \int_0^u \boldsymbol{\eta}(u, 0) du \quad (2)$$

where  $\mathbf{x}_0 = [x_0 \ y_0 \ z_0]^T$  is the position vector at the end point  $P(0, 0)$ .

Next, we consider description of the deformed surface of a belt object. According to differential geometry, the normal curvature  $\kappa$  in direction  $\mathbf{d} = a\boldsymbol{\eta} + b\boldsymbol{\zeta}$  of the object surface is represented as follow:

$$\kappa = \frac{La^2 + 2Mab + Nb^2}{Ea^2 + 2Fab + Gb^2} \quad (3)$$

where  $E$ ,  $F$ , and  $G$  are coefficients of first fundamental form and  $L$ ,  $M$ , and  $N$  are those of the second fundamental form of the surface. These coefficients are defined as follows:

$$E = \boldsymbol{\eta} \cdot \boldsymbol{\eta} = 1, \quad F = \boldsymbol{\eta} \cdot \boldsymbol{\zeta} = 0, \quad G = \boldsymbol{\zeta} \cdot \boldsymbol{\zeta} = 1 \quad (4)$$

$$L = \frac{\partial \boldsymbol{\eta}}{\partial u} \cdot \boldsymbol{\zeta} = -\omega_\zeta, \quad M = \frac{\partial \boldsymbol{\zeta}}{\partial u} \cdot \boldsymbol{\xi} = \omega_\eta, \quad N = \frac{\partial \boldsymbol{\zeta}}{\partial v} \cdot \boldsymbol{\xi}. \quad (5)$$

Here, we introduce parameter  $\delta(u, 0) = N$ . It corresponds to the curvature in the transverse direction.

The normal curvature  $\kappa$  depends on the direction  $\boldsymbol{d}$  and its maximum value  $\kappa_1$  and its minimum value  $\kappa_2$  are called the principal curvature. Direction  $\boldsymbol{d}_1$  of the maximum curvature  $\kappa_1$  and direction  $\boldsymbol{d}_2$  of the minimum curvature  $\kappa_2$  are referred to as principal directions. A surface is characterized by Gaussian curvature  $K(u, 0)$  and the mean curvature  $H(u, 0)$ . They are related to the principal curvature  $\kappa_1$  and  $\kappa_2$  by

$$K = \kappa_1 \kappa_2 = -\omega_\zeta \delta - \omega_\eta^2 \quad (6)$$

$$H = \frac{\kappa_1 + \kappa_2}{2} = \frac{\delta - \omega_\zeta}{2} \quad (7)$$

Thus, bending of a surface is characterized by infinitesimal ratio of rotation angle  $\omega_\eta(u, 0)$  and  $\omega_\zeta(u, 0)$  and the curvature in a transverse direction  $\delta(u, 0)$ .

If a principal curvature  $\kappa_2$ , i.e., the minimum value of the normal curvature is equal zero, the surface is developable. Namely, it can be flattened without its expansion or contraction. Such surface is referred to as a developable surface. In this paper, we assume that a belt object is inextensible. Then the deformed shape of the object corresponds to a developable surface. Gaussian curvature  $K$  of a developable surface must be zero at any point. So, the following constraint is imposed on the object.

$$K = -\omega_\zeta \delta - \omega_\eta^2 = 0, \quad \forall u \in [0, U]. \quad (8)$$

This implies that  $\delta$  can be described by

$$\delta = -\frac{\omega_\eta^2}{\omega_\zeta}. \quad (9)$$

The infinitesimal ratio of rotation angle around  $\xi$ -axis  $\omega_\xi$  must also be satisfied the following equation because of the inextensibility of a belt object:

$$\omega_\xi(u) = 0, \quad \forall u \in [0, L]. \quad (10)$$

## POTENTIAL ENERGY AND GEOMETRIC CONSTRAINTS

Let us formulate the potential energy of a deformed belt object with Kirchhoff theory[14]. In this paper, we supposed the deformation of a belt object as follows:

1. Straight lines normal to the mid-surface remain straight after deformation.
2. Straight lines normal to the mid-surface remain normal to the mid-surface after deformation.
3. The thickness of the plate does not change during a deformation.

We can formulate the potential energy of a belt object with Kirchhoff theory. That is described as follows:

$$U = \frac{1}{2} \int_0^L \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} [\sigma_\xi \varepsilon_\xi + \sigma_\eta \varepsilon_\eta + \sigma_\zeta \varepsilon_\zeta + \tau_{\xi\eta} \gamma_{\xi\eta} + \tau_{\eta\zeta} \gamma_{\eta\zeta} + \tau_{\xi\zeta} \gamma_{\xi\zeta}] du dv dw \quad (11)$$

$$= \frac{E}{2(1-\nu^2)} \frac{bh^3}{12} \int_0^L [\kappa_\eta^2 + \kappa_\zeta^2 + 2\nu\kappa_\eta\kappa_\zeta] du + \frac{G}{2} \frac{bh^3}{3} \int_0^L [\kappa_{\eta\zeta}^2] du$$

where  $E$ ,  $G$  and  $\nu$  represents Young's modulus, modulus of rigidity and Poisson ratio, respectively. Poisson ratio  $\nu$  is zero because the deformed shape of the object is developable.  $\kappa_\eta$ ,  $\kappa_\zeta$ , and  $\kappa_{\eta\zeta}$  are curvature in the  $\eta$ -direction, the  $\zeta$ -direction and ratios of rotation angle around the  $\eta$ -axis. These correspond to  $-\omega_\zeta$ ,  $\delta$ , and  $\omega_\eta$  respectively. So, Eq. (11) is described by

$$U_1 = \frac{E}{2} \frac{bh^3}{12} \int_0^L [\omega_\zeta^2 + (-\frac{\omega_\eta^2}{\omega_\zeta})^2] du + \frac{G}{2} \frac{bh^3}{3} \int_0^L [\omega_\eta^2] du. \quad (12)$$

Next, let us formulate geometric constraints imposed on a belt object. Let  $\boldsymbol{l} = [l_x \quad l_y \quad l_z]^T$  be a predetermined vector describing the relative position between two operational points on the central axis of a belt object,  $P(u_a)$  and  $P(u_b)$ . Recall that the spatial coordinates corresponding to distance  $u$  are given by Eq.(2). Thus, the following equation must be satisfied:

$$\boldsymbol{x}(u_b) - \boldsymbol{x}(u_a) = \boldsymbol{l} \quad (13)$$

The oriental constraint at operation point  $P(u_a)$  is simply described as follows:

$$\boldsymbol{\xi}(u_c) = \boldsymbol{\xi}_c, \boldsymbol{\eta}(u_c) = \boldsymbol{\eta}_c, \boldsymbol{\zeta}(u_c) = \boldsymbol{\zeta}_c \quad (14)$$

where  $\boldsymbol{\xi}_c$ ,  $\boldsymbol{\eta}_c$  and  $\boldsymbol{\zeta}_c$  are predefined unit vectors at this point. Therefore, the shape of a belt object is determined by minimizing the potential energy described by Eq.(12) under geometric constraints imposed on the object described by Eqs.(13) and (14). Namely, computation of the deformed shape of the object results in variational problem under equation and inequality constraints.

## COMPUTATIONAL ALGORITHM FOR BELT OBJECT MANIPULATION

Wakamatsu developed an algorithm based on Ritz's method [15] and a nonlinear programming technique to compute linear object deformation. That algorithm can be applied to the computation of the belt object deformation.

Let us express function  $\omega_\zeta(u)$  and  $\omega_\eta(u)$  by linear combinations of basis function  $e_1(u)$  through  $e_n(u)$ :

$$\omega_\zeta(u) = \boldsymbol{a}^\zeta \cdot \boldsymbol{e}(u), \quad \omega_\eta(u) = \boldsymbol{a}^\eta \cdot \boldsymbol{e}(u) \quad (15)$$

where  $\boldsymbol{a}^\zeta$  and  $\boldsymbol{a}^\eta$  are vectors consisting of coefficients corresponding to functions  $\omega_\zeta(u)$  and  $\omega_\eta(u)$  respectively, and vector  $\boldsymbol{e}(u)$  is composed of basis function  $e_1(u)$  through  $e_n(u)$ . Substituting the above equation into Eq.(12), potential energy  $U_1$  is described by a function of coefficient vectors  $\boldsymbol{a}^\zeta$  and  $\boldsymbol{a}^\eta$ . Constraints are also described by conditions involving the coefficient vectors. Consequently, the deformed shape of a belt object can be derived by computing a set of coefficient vectors

$\mathbf{a}^\zeta$  and  $\mathbf{a}^\eta$  that minimizes the potential energy under the constraints. The minimizing problem can be solved by use of a nonlinear programming technique such as multiplier method[16].

Let us compute the belt object deformation. The natural shape of the belt object is illustrated in Fig.3-(a). It is 200mm long, 20mm wide, and 0.14mm thick. Positional and orientational constraints are shown in Fig.3-(b). Fig.4 shows the computational results. In this approach, computation was performed on two 2.0GHz AMD Operation 246 CPUs with 3GB memory operated by Solaris 10. Program were compiled by a Sun C Compiler 5.8 with optimization option O5. It took about 10 minutes to compute.

## DEFORMATION PATH PLANNING FOR BELT OBJECT DEFORMATION

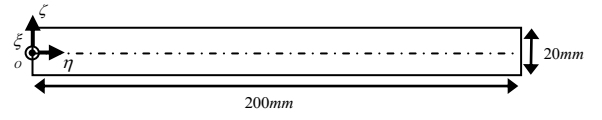
In manipulation of a deformable belt object, the object is often deformed from one shape into another. Let us determine appropriate deformation path from an initial shape to a goal shape. It is generally required to deform a belt object with little damage. Excessive potential energy of a belt object can be easily transformed into kinetic energy by a small disturbance force, in which case the shape of the object may become unstable and change dynamically. Thus, the potential energy of a belt object should be small during its deformation process. It is found that a deformable path that minimizes the value of the maximum potential energy is preferable. Recall that the deformation of a belt object can be described by coefficient vectors corresponding to  $\omega_\zeta(u)$  and  $\omega_\eta(u)$ . Let  $\mathbf{a}$  be a collective vector of these coefficients. One deformation corresponds to a point in coefficient space. The deformation process of a belt object is then given by a path in the coefficient space. Let  $\mathbf{a}_0$  and  $\mathbf{a}_1$  be the initial and goal deformations, respectively, and let  $\mathbf{a}(k)$  ( $0 \leq k \leq 1$ ) be a path from the initial deformation to the goal deformation. Note that functions  $1-k, k, k^i(1-k)$  ( $i = 1, 2, \dots$ ), and  $k(1-k)^j$  ( $j = 1, 2, \dots$ ) are a set of bases of a function space. Then, any path can be approximately by a linear combination of these base functions

$$\mathbf{a}(\mathbf{b}, k) = (1-k)\mathbf{a}_0 + k\mathbf{a}_1 + \sum_{i=1}^{\infty} \mathbf{b}_{i,0} k^i (1-k) + \sum_{j=1}^{\infty} \mathbf{b}_{j,1} k (1-k)^j \quad (16)$$

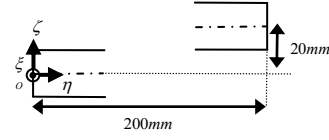
where  $\mathbf{b}_{i,0}, \mathbf{b}_{j,1}$  are expansion coefficients. Any path can be represented by an infinite number of coefficients vectors:  $\mathbf{b}_{i,0}, \mathbf{b}_{j,1}$ . Let  $\mathbf{b}$  be a collective vector consisting of these coefficient vectors, which is referred to as *the deformation path vector*. The deformation path vector  $\mathbf{b}$  determines a deformation path from the initial deformation  $\mathbf{a}(\mathbf{b}, 0) = \mathbf{a}_0$  to the goal deformation  $\mathbf{a}(\mathbf{b}, 1) = \mathbf{a}_1$ . Vectors  $\mathbf{a}(\mathbf{b}, k)$  corresponds to an intermediate deformation along the path.

Let  $U(\mathbf{b}, k)$  be the potential energy of a belt object with deformation  $\mathbf{a}(\mathbf{b}, k)$ . Let  $U_{max}(\mathbf{b})$  be the maximum of the potential energy along a deformation path represented by  $\mathbf{b}$ :

$$U_{max}(\mathbf{b}) = \max_{0 \leq k \leq 1} U(\mathbf{b}, k). \quad (17)$$

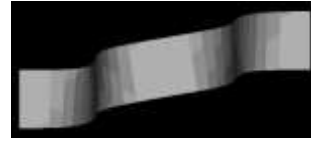


(a) Initial shape

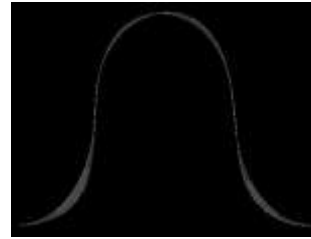


(b) Deformed shape

Figure3: Example of belt object deformation



(a) Top view



(b) Front view



(c) Side view

Figure4: Computational result

Recall that geometric constraints imposed on an object can be described by a set of functions of vector  $\mathbf{b}$ . Consequently, it is found that the optimal deformation path can be derived by minimizing the function  $U_{max}(\mathbf{b})$  under the geometric constraints.

Let us show a numerical example in order to demonstrate how deformation path is computed by our approach. The natural shape of the object is illustrated in Fig.3-(a). Both ends of the object are fixed as shown in Fig.3-(b) and the object is deformed from being convex upward to being convex downward. Fig.5 shows the computed optimal deformation path. Fig5-(a) shows the initial shape of the belt object and Fig.5-(f) shows the goal shape of it. It took about 2 hours to compute the deformation path.

## EXTENSIBLE MODEL OF BELT OBJECT

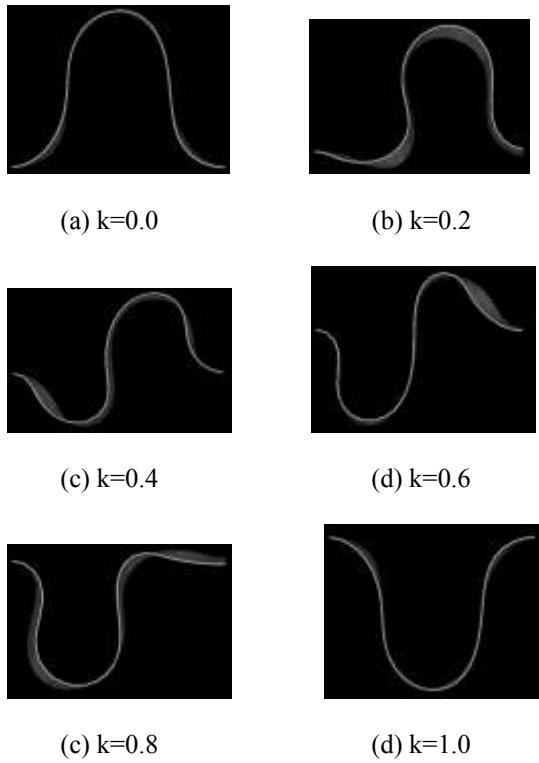


Figure5: Deformation path for a belt object

In this paper, we assumed that a belt object bends and twists mainly and their expansion/contraction can be negligible. So, we assumed that the surface of the belt object is developable surface and Gaussian curvature of the object is zero at any points. However, generally, an elastic object can expand and contract during its bending or twisting. The surface of the deformed object with expansion/contraction is not developable.

In manipulation, the force that causes the object expansion/contraction usually are imposed. When the object is extensible, it expands or contracts. However, when the object is inextensible, such as film/flexible circuit boards, this force can lead to locally excessive stress in the object, instead of expansion/contraction. When we evaluate the damage of a belt object with the whole potential energy as previously explained, the locally excessive stress less contributes to the damage of the object. However, locally excessive stress can lead to fracture in a belt object. So, the locally excessive stress should be small during its deformation process.

In this paper, the deformation involving expansion/contraction is referred as *undevelopable deformation*. When the surface of a belt object is developable, the potential energy of a deformed belt object is described as Eq.(12). Here, let us formulate the potential energy of a belt object that deforms *undevelopably*. In this paper, the model where the object surface is assumed to be undevelopable is referred as *extensible model* while the model explained in Section 2 is referred as *inextensible model*. When a belt object deforms undevelopably, the surface of the belt object is undevelopable and Gaussian curvature of the object isn't zero at any points. Furthermore, the central axis can bend

around  $\xi$ -axis. The potential energy by this deformation is described as follows:

$$U_{\xi} = \frac{E}{2(1-\nu^2)} \frac{b^3 h}{12} \int_0^L [\omega_{\xi}^2] du. \quad (18)$$

In addition, when a belt object deforms undevelopably, it expands and contracts during its bending or twisting. Namely, Poisson ratio  $\nu$  isn't zero in Eq.(11). So, the potential energy of the shape including the undevelopable deformation is described as follows:

$$U_2 = \frac{E}{2(1-\nu^2)} \frac{bh^3}{12} \int_0^L [\omega_{\xi}^2 + \delta^2 - 2\nu\omega_{\xi}\delta] du + \frac{G}{2} \frac{bh^3}{3} \int_0^L [\omega_{\eta}^2] du. \quad (19)$$

$$+ \frac{E}{2(1-\nu^2)} \frac{b^3 h}{12} \int_0^L [\omega_{\xi}^2] du.$$

Then, modulus of rigidity  $G$  can be described as follows:

$$G = \frac{E}{2(1+\nu)} \quad (20)$$

Expressing function  $\omega_{\xi}(u)$ ,  $\omega_{\eta}(u)$ ,  $\omega_{\zeta}(u)$ , and  $\delta(u)$  by linear combinations of basis function  $e_1(u)$  through  $e_n(u)$  as previously explained, potential energy  $U_2$  is described by a function of coefficient vectors  $\mathbf{a}^{\xi}$ ,  $\mathbf{a}^{\eta}$ ,  $\mathbf{a}^{\zeta}$ , and  $\mathbf{a}^{\delta}$ . Constraints are also described by conditions involving the coefficient vectors. Consequently, the deformed shape of a belt object can be derived by computing a set of coefficient vectors  $\mathbf{a}^{\xi}$ ,  $\mathbf{a}^{\eta}$ ,  $\mathbf{a}^{\zeta}$ , and  $\mathbf{a}^{\delta}$  that minimizes the potential energy under the constraints. Fig.6 shows a computational result of a belt object deformation with extensible model. It seems that the belt object shown in Fig.6 twists more largely than that in Fig.4. This is because when Poisson ratio  $\nu$  isn't zero, modulus of rigidity  $G$  becomes small and the potential energy with respect to  $\omega_{\eta}(u)$  less contributes to the whole energy.

Moreover, let us introduce the extensible model into the deformation path planning. Let  $\mathbf{a}$  be a collective vector of coefficients vectors  $\mathbf{a}^{\xi}$ ,  $\mathbf{a}^{\eta}$ ,  $\mathbf{a}^{\zeta}$ , and  $\mathbf{a}^{\delta}$ , and define  $\mathbf{a}(\mathbf{b}, k)$  and  $U_{max}(\mathbf{b})$  as represented Eqs.(16) and (17). Consequently, the new optimal deformation path can be derived by minimizing the function  $U_{max}(\mathbf{b})$  with the same algorithm previously mentioned. Fig.7 shows the computational result of new optimal deformation path. It seems that the shapes in the deformation path obtained with extensible model are also more twisted.

## COMPARISON OF TWO MODELS

In this section, the computational results obtained with two models will be verified. The model that can be developable surface is referred as *inextensible model*. First, the computational results obtained with *inextensible model* and *extensible model* will be verified by measuring the deformed shape of the belt object. We measured a belt-shaped flexible polystyrol sheets with 3D scanner. It is 200mm long, 20mm wide, and 0.14mm thick. Young's modulus  $E$  is 3.0GPa and Poisson ratio  $\nu$  is 0.34. Fig.8 shows the experimental result. Fig.9-(a) and Fig.9-(b) show the computational results obtained with *inextensible model* and *extensible model* respectively. As



(a) Top view



(b) Front view



(c) Side view

Figure6: Computational result with extensible model



(a) k=0.0



(b) k=0.2



(c) k=0.4



(d) k=0.6



(c) k=0.8



(d) k=1.0

Figure7: Deformation path for a belt object with extensible mode

shown in these figures, both computational results are well coincide with the actual shape. This means that, actually, the surface of a belt object is nearly developable surface and a belt object bends and twists mainly and their expansion/contraction can be negligible. Second, in both shapes shown in Fig.9-(a) and Fig.9-(b), the locally stress is verified, and the point which is subjected to maximum stress is calculated. The strain in a belt object is represented with the curvature  $\omega_\xi(u)$ ,  $\omega_\eta(u)$ ,  $\omega_\zeta(u)$ , and  $\delta(u)$  as follows:

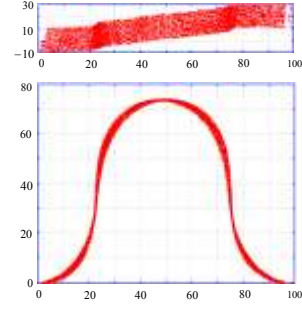
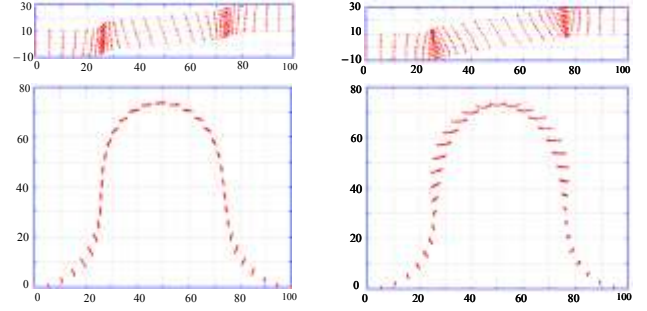


Figure8: Experimental result



(a) inextensible model

(b) extensible model

Figure9: Computational result

$$\begin{bmatrix} \varepsilon_u(u, w, v) \\ \varepsilon_v(u, w, v) \\ \gamma_{uv}(u, w, v) \end{bmatrix} = \begin{bmatrix} \omega_\xi/2 & -\omega_\xi/2 \\ 0 & \delta/2 \\ 0 & \omega_\eta \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \quad (21)$$

As mentioned before,  $u$  is the distance from one end of the object along the central axis in its longitudinal direction,  $v$  is the distance from the central axis in the transverse direction of the object, and  $w$  be the distance from the neutral plane in the vertical direction. The stress in a belt object is described by

$$\begin{bmatrix} \sigma_u(u, w, v) \\ \sigma_v(u, w, v) \\ \tau_{uv}(u, w, v) \end{bmatrix} = \frac{2G}{1-2\nu} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & (1-2\nu)/2 \end{bmatrix} \begin{bmatrix} \varepsilon_u \\ \varepsilon_v \\ \gamma \end{bmatrix} \quad (22)$$

Moreover, the principal stress is represented as follows:

$$\sigma_{1,2} = \frac{1}{2}(\sigma_u + \sigma_v) \pm \frac{1}{2}\sqrt{(\sigma_u - \sigma_v)^2 + 4\tau_{uv}^2} \quad (23)$$

where  $\sigma_1$  is the maximum principal stress and  $\sigma_2$  is the minimum principal stress. Table 1 shows the value of the maximum principal stress of the deformed object shown in Fig.9-(a) and Fig.9-(b) and the position where the maximum principal stress is occurred. In both cases, the position where the maximum principal stress is occurred is the same position. However, the maximum principal stress of the deformed shape obtained with extensible model is larger than that of the deformed shape obtained with inextensible model. This is because a belt object can deform undevelopably in the

Table1. Principal stress

	Principal stress [MPa]	Position (u,v,w) [mm]
Inextensible Model	10.7	(100, -10, -0.07)
Extensible Model	13.0	(100, -10, -0.07)

extensible model and undevelopable deformation often causes the locally excessive stress in a belt object.

## FUTURE WORKS

It is found that undevelopable deformation causes the locally excessive stress in a belt object. The locally excessive stress can make fracture. So, the undevelopable deformation should be small during its deformation process. It is assumed that when the potential energy  $U_2$  obtained with extensible model is nearly equal to  $U_1$  obtained with inextensible model, a belt object less deforms undevelopably and the surface of the deformed belt object becomes almost developable. So, new potential energy  $U_{new}$  can be defined as follow:

$$U_{new} = c_1 U_2 + c_2 |U_1 - U_2| \quad (24)$$

where  $c_1$  and  $c_2$  are the weighting factors of whole potential energy and undevelopable deformation respectively. The deformation path of a belt object is determined by minimizing the new potential energy  $U_{new}$  under geometric constraints. As a result, a belt object deforms developably and the stress in the object becomes small during its manipulation process.

## CONCLUSION

A differential geometry based modeling to represent belt object deformation and optimal deformation path were proposed for manipulation of film/flexible circuit boards. First, the deformed shape of a belt object was assumed to be developable surface and represented as functions of two independent parameters. Then we estimated belt object deformation by optimizing these parameters so that potential energy of the object attains its minimum value under constraints imposed on it. Furthermore, adequate deformation path was derived by minimizing maximum potential energy during its deformation process. Second, undevelopably deformed shape and deformation path were derived. Third, the deformed shape and the maximum principal stress in the objects obtained with inextensible model and extensible model were compared, and it was verified that undevelopable deformation causes the excessive stress in a belt object. Finally, as future prospects, a method to generate the path planning was proposed so that the deformed shape of a belt object becomes developable.

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