A MODEL-BASED GENERATION OF FIXTURE LAYOUT CANDIDATES FOR WORKPIECE HOLDING

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ABSTRACT

A systematic approach to the generation of fixture location layouts is presented. Workpiece holding is one of difficult processes to be automated. Expert technicians have a capability of determining a fixture layout appropriate to the given workpiece and the given operation. Fixture planning system must have this capability in order to automate the holding process.

In this article, we will develop a computational procedure that can derive fixture layouts for various workpieces. Firstly, fixture layouts are represented by topological relationship between the workpiece and a set of fixtures. Secondly, static properties at each layout are formulated by use of the cone theory. Next, a necessary condition for reliable clamping is derived and a procedure to investigate this condition based on the model of the workpiece is developed using the cone theory. Finally a simple example is shown to demonstrate the computation process of possible fixture layouts.

INTRODUCTION

Workpiece holding is one of difficult processes to be automated. In factories, most holding processes are performed by technicians whereas machining processes are done by machines. Automatic holding of workpieces is thus required eagerly. Recently, some researches to automatic fixture planning have been studied (Chang, 92). Workpieces must be fixed to tables and pallets firmly so that the machining process should be achieved successfully. Skillful technicians have a capability of determining an adequate method of clamping from the geometrical shape and the material of workpieces and machining conditions such as feed rate and cutting forces. Fixture planning system must have this capability of determining a holding method.

Some approaches to fixture planning have been developed in order to automate the determination process by expert technicians. One approach to the fixture planning is based on expert systems (Liou and Suen, 92). Empirical knowledge of expert technicians is extracted and is expressed as a set of production rules. Clamping methods can be then inferred from the information of workpieces and fixtures by applying the production rules. This approach has, however, the following drawback; it is difficult to acquire the rules from expert technicians since the technicians often cannot explain their decision. Another approach is an analytical one based on the model of workpieces and fixtures (Nguyen, 86). Location of fixtures and clamping forces exerted by the fixtures are derived analytically from the model of workpieces and fixtures built on a computer. This approach is so systematic that it can be applied to various workpieces once the model of clamping process is developed sufficiently.

There exist many fixture layouts where the topological and the geometrical relationships between a workpiece and fixtures differ from one another. For the analytical fixture planning, a computational method to find fixture layouts capable of holding the workpiece should be established. In this article, we will develop an analytical method to enumerate possible topological layouts of fixtures. Firstly, fixture layouts are described by listing all contacting pairs between a workpiece and a set of fixtures. Secondly, static behavior at each fixture layout is analyzed and is formulated by use of the theory of polyhedral convex cones. Next, a necessary condition for reliable holding is derived based on the static formulation. Computational procedure to investigate this condition is also developed using the cone theory. Finally, a simple example is shown to demonstrate the computation process of fixture layouts by the proposed approach.

REPRESENTATION OF FIXTURE LAYOUTS

Fixture Layouts

Workpiece holding is a process of locating and clamping a workpiece by fixtures. The process mainly consists of three subprocesses; placing process, guiding process, and clamping process. In placing process, some guiding fixtures are placed to appropriate locations in a workspace. In guiding process, a workpiece is guided along the located fixtures to the goal configuration. The workpiece is required to be guided to the goal configuration easily and to be positioned precisely at that configuration. In the clamping process, clamping fixtures are located to hold the workpiece rigidly. The workpiece must be held by fixtures without causing excess deformation of the workpiece. Fixture planning is a process to determine a fixture location appropriate to holding a workpiece. The fixture planning requires the geometrical shape and the material of the workpiece and the machining condition. Experienced technicians are capable of planning an appropriate fixture location for various workpieces.

There exist many fixture layout where the topological and the geometrical relationships between a workpiece and a set of fixtures are different from one another. Figure 1 illustrates some examples of possible fixture layout for a planar workpiece by point-contact fixtures. The figure illustrates topologically different layouts of fixtures. For example, Figure 1-(a) shows a fixture layout where edge 1 and edge 3 are in contact with point fixtures. The workpiece is not fixed by this layout. Figure 1-(b) expresses another layout where edge 2 and edge 3 are in contact with the fixtures. Note that the workpiece can be held when the two fixture are located opposite to each other. Namely, this layout has a possibility to holding the workpiece rigidly. In the fixture planning, both the topological relationship and the geometrical features must be determined. One approach to the determination is that the possible topological layouts are firstly enumerated and one layout is then chosen after deciding the geometrical features for individual layout candidates. All of the fixture layout candidates which have a possibility to clamping the workpiece should be enumerated in order to establish the above systematic approach to the fixture planning. Thus, we will focus on the enumeration of fixture layout candidates in this article.

Contact Pairs

Workpiece holding is performed by mechanical contacts between a workpiece and fixtures. The force and the moment applied to a workpiece by contacting fixtures strongly depend upon the geometrical shape of the fixtures and their layout as well as the friction at the contacting points. Let us express a topological fixture layout by the pairs between workpiece elements and fixture elements. Let us consider the following work-

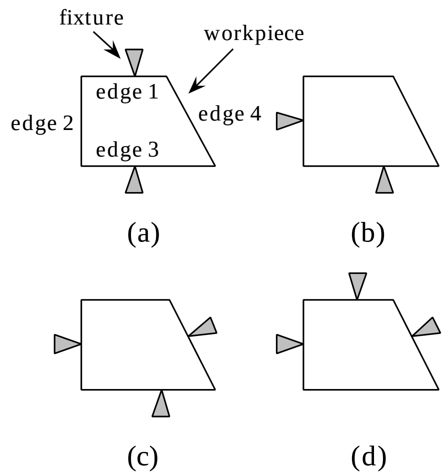


Figure 1. Simple example of fixture layout

piece elements; 1) faces, 2) edges, and 3) vertices. Assume that fixture elements consist of a) point-contact fixtures, b) line-contact fixtures, and c) plain-contact fixtures. The contacting pairs are then expressed by a pair between a workpiece element and a fixture element. Mechanical contacts between a workpiece and fixtures are represented by listing contacting pairs. The configuration of each fixture can be described by the relative position and orientation between the fixture and its corresponding workpiece element. Let $\boldsymbol{q}_i$ be a vector representing the relative configuration between the workpiece and the i-th fixture. Assuming that the workpiece is fixed by n fixtures, the relative configuration between a workpiece and a set of fixtures can be given by the collective vector $\boldsymbol{q}$ consisting of vector $\boldsymbol{q}_1$ through $\boldsymbol{q}_n$ as follows:

$$\boldsymbol{q} = [\boldsymbol{q}_1, \boldsymbol{q}_2, \cdots, \boldsymbol{q}_n]. \tag{1}$$

Namely, the fixture location is expressed by the configurations corresponding to the individual contact pairs.

Since workpiece elements and fixture elements consist of bounded faces, the range of configuration $\boldsymbol{q}_i$ is also bounded. Let D_i be a set of relative configurations between the workpiece and the i-th fixture. For example, let us consider a contact pair between a point-contact fixture and a planar face. The position of the fixture must be within the face and the orientation of the fixture must be chosen so that the fixture are not interfered with by the workpiece. The position and the orientation of the fixture thus must be limited and the relative configuration is bounded in a certain region. Note that a boundary representation model of solid objects provides all information necessary to compute set D_i. For example, the range of the position of a point fixture contacting to a workpiece face is derived by the coordinates of all end points of the face. The orientation of the fixture is determined by the outward normal vector of the face. Thus, the configuration set D_i corresponding to a contact pair between a point-contact fixture and a workpiece face can be computed from the geometrical features of the workpiece face. Consider a contact pair between a line fixture and an edge of the workpiece. The range of the fixture position is determined by the end points of the line fixture and those of the edge. The configuration set corresponding to a contact pair between a line fixture and a workpiece edge can be calculated from the geometrical features of the line fixture and the workpiece edge. Generally, set D_i can be computed numerically from the geometrical model of a workpiece and that of fixtures on a computer.

STATIC ANALYSIS OF CLAMPING PROCESS

Static Modeling of Workpiece Holding Using Cone Theory

In this section, we formulate static relationship of a workpiece constrained by fixtures. An efficient mathematical tool have been established based on the theory of *Polyhedral Convex Cones* (Goldman and Tucker, 56) in order to deal with unidirectional constraints due to mechanical contacts (Hirai and Asada, 93). This tool provides an efficient formalization for treating unidirectional constraints that we need to deal with in a fixture process. In the following analysis, we will investigate statically admissible force and mement $\boldsymbol{p}$. Vector $\boldsymbol{p} = [f_x, f_y, f_z, m_x, m_y, m_z]$ is the force and moment acting on the workpiece that satisfy the static equilibrium condition.

Fixture layout is described by a list of contact pairs, as mentioned before. Each contact pair causes a reaction force upon the workpiece. Let $\boldsymbol{d}_i$ be the wrench vector corresponding to the i-th contact pair (Ohwovoriole and Roth, 81). If all of the contact points are frictionless, the force and moment equivalent to the reaction force at the i-th contact pair is given by $r_i \boldsymbol{d}_i$, where coefficient r_i denotes the magnitude of the reaction force. Note that coefficient r_i is non-negative since the contacts are unidirectional. The range of reaction forces is thus described by the sum of equivalent forces. Namely,

$$F = \{\sum_{i=1}^{n} r_i \boldsymbol{d}_i \mid r_i \geq 0,\ i \in [1, n]\} \tag{2}$$

where $[1, n]$ represents integers 1 through n. Set F is a semi-infinite cone in the 6-dimensional space consisting of a finite number of edges, as shown in Figure 2. This set is called a polyhedral convex cone and is abbreviated to PCC. Vector $\boldsymbol{d}_i$ is a vector along an edge of the cone. Let us express the above equation simply by

$$F = span\{\boldsymbol{d}_1, \boldsymbol{d}_2, \cdots, \boldsymbol{d}_n\}. \tag{3}$$

This form is referred to as the *Span Form* of the PCC. Each vector involved is called a span vector. Therefore, the reaction force set is described by a PCC.

As shown in Figure 2, set F can be regarded as a cone surrounded by a finite number of hyperplanes. Let $\boldsymbol{a}_1$ through $\boldsymbol{a}_h$ be outer normal vectors to the hyperplanes.

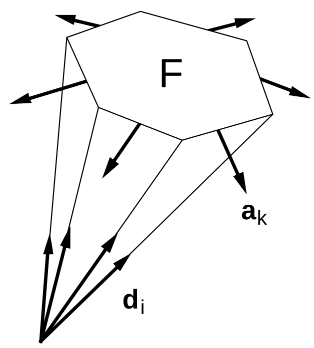


Figure 2. Polyhedral convex cone

Set F is then expressed as follows:

$$F = \{\boldsymbol{p} \mid \boldsymbol{a}_j^T \boldsymbol{p} \leq 0,\ \forall j \in [1, h]\}. \tag{4}$$

For the sake of simplicity, the set given by eq.(4) is denoted as

$$F = face\{\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_h\} \tag{5}$$

which is referred to as the *Face Form* of the PCC. Each vector involved is called a face vector.

Let us consider friction the between the workpiece and the fixtures. Friction is represented by the *friction cone* (Erdmann, 86), which specifies the range of reaction forces. The axis of the cone is parallel to the normal vector of the surface. Sides of the cone make an angle $\tan^{-1} \mu$, where μ denotes the coefficient of friction. Let us approximate the friction cone by a PCC consisting of m_i span vectors. The force and moment equivalent to the reaction forces at the i-th contact pair is then given by a PCC in the span form as follows:

$$F_i = span\{\ \boldsymbol{d}_{ik} \mid k \in [1, m_i]\}. \tag{6}$$

The collective reaction force is described by the convex sum of the friction cones at individual contact pair. The set of reaction forces is thus given by

$$F = F_1 + F_2 + \cdots + F_n \tag{7}$$

where $+$ denotes the convex sum of two sets. The convex sum of sets X and Y is defined as follows:

$$X + Y \stackrel{\triangle}{=} \{\boldsymbol{x} + \boldsymbol{y} \mid \forall \boldsymbol{x} \in X, \forall \boldsymbol{y} \in Y\}. \tag{8}$$

It has been shown that the convex sum of PCC's is also given by a PCC and its span form consists of the span vectors of individual PCC's (Goldman and Tucker, 56). This implies that the span form of cone F is given by span vectors of cones F_1 through F_n. Namely,

$$F = span\{\ \boldsymbol{d}_{ik} \mid i \in [1, n],\ k \in [1, m_i]\}. \tag{9}$$

Thus, the set of reaction forces is described by a polyhedral convex cone in the span form as well.

Force Closure Clamping

In workpiece holding, fixtures apply unidirectional forces and moments upon the workpiece. A desired condition for the holding is to guarantee that no motion occurs no matter what disturbance force and moment are imposed on the workpiece. This condition, which is referred to as *Force Closure*, has been given by Ohwovoriole and Roth (Ohwovoriole and Roth, 81). The condition can be restated by using the reaction force set given by eq.(7). An arbitrary non-zero force $\boldsymbol{p}$ which is not involved in the reaction force set F violates the force equilibrium condition and causes some workpiece motion. The force closure condition is not satisfied in this case. Thus, for the force closure clamping, the reaction force set must involve all the forces and moments in the 6-dimensional vector space R^6:

$$F = R^6 \tag{10}$$

The problem of force closure is basically to investigate whether the set F covers the whole vector space or not. The above equation can be examined by use of the computational algorithms of polyhedral convex cones (Hirai and Asada, 93).

COMPUTATION OF FIXTURE LAYOUT CANDIDATES

Possible Reaction Force Sets

The set of the force and the moment that can be applied to a workpiece by each fixture varies according to the relative configuration between the workpiece and the fixture. For example, consider a contact pair between a planar face of the workpiece and a point-contact fixture. The moment acting on the workpiece varies according to the location of the contacting point while the direction of the translational force does not change. For a contact pair between a vertex element of the workpiece and a planar-face fixture, the force set depends upon the direction of the planar fixture. As shown in eq.(6), the set of reaction forces is described by a PCC. The span vectors of the cone depend upon the relative configuration, that is, the force cone corresponding to the i-th contact pair at configuration $\boldsymbol{q}_i$ is described as follows:

$$F_i(\boldsymbol{q}_i) = span\{\, \boldsymbol{d}_{ik}(\boldsymbol{q}_i) \mid k \in [1, m_i]\}. \tag{11}$$

Assume that the workpiece is clamped by n fixtures. As shown in eq.(7), the reaction force set F at configuration $\boldsymbol{q} = [\boldsymbol{q}_1, \cdots, \boldsymbol{q}_n]$ is given by

$$\begin{aligned} &F(\boldsymbol{q}_1, \boldsymbol{q}_2, \cdots, \boldsymbol{q}_n) = \\ &F_1(\boldsymbol{q}_1) + F_2(\boldsymbol{q}_2) + \cdots + F_n(\boldsymbol{q}_n). \end{aligned} \tag{12}$$

Recall that a fixture location satisfies the force closure condition if the force set F covers the whole vector space. Location of fixtures is characterized by configurations $\boldsymbol{q}_1$ through $\boldsymbol{q}_n$ corresponding to the individual fixtures. The condition for force closure clamping is thus given by the following equation:

$$\exists \boldsymbol{q}_1, \boldsymbol{q}_2, \cdots, \boldsymbol{q}_n \ \text{ s.t. } \ F(\boldsymbol{q}_1, \boldsymbol{q}_2, \cdots, \boldsymbol{q}_n) = R^6. \tag{13}$$

Span vectors of reaction force set $F_i(\boldsymbol{q}_i)$ are nonlinear functions with respect to configuration $\boldsymbol{q}_i$. In addition, the force closure condition can be checked by applying numerical procedures rather than by solving analytical equations. In order to investigate whether the above condition is held or not, it is required to check the force closure condition for all combination of the configurations $\boldsymbol{q}_1$ through $\boldsymbol{q}_n$. Thus, it is difficult to examine whether the above condition is satisfied or not in an analytical method nor in a numerical computation approach.

Instead of investigating the above condition, we will examine a necessary condition for the force closure clamping. The difficulty of the above condition results from the dependency of set F_i upon configuration $\boldsymbol{q}_i$. In order to evaluate possible reaction force corresponding to each pair collectively, let us introduce a collection of reaction force sets for all configurations at each contact pair. An aggregation of the force sets corresponding to the i-th contact pair is then defined by

$$P_i \triangleq \bigcup_{\boldsymbol{q}_i \in D_i} F_i(\boldsymbol{q}_i) \tag{14}$$

This set is referred to as *Possible Reaction Force Set* corresponding to the i-th contact pair. Since the reaction force set $F_i(\boldsymbol{q}_i)$ at an arbitrary configuration $\boldsymbol{q}_i$ is involved in the possible reaction force set P_i, if the force closure fixture layout condition given by eq.(13) is satisfied, the following condition is held:

$$P = R^6 \tag{15}$$

where set P is the convex sum of possible reaction force sets corresponding to individual contact pairs:

$$P = P_1 + P_2 + \cdots + P_n. \tag{16}$$

If the force closure condition given by eq.(13) is satisfied, the above equation is held. Namely, the above equation is a necessary condition for a force closure fixture layout. Force closure condition given by eq.(13) is not always satisfied even if the above equation is held. Nevertheless, the above equation is effective since sets P_1 through P_n are independent of configuration $\boldsymbol{q}$. Set P denotes the possible reaction force and moment by a set of fixtures. Note that set P is described by a convex sum of possible reaction force sets P_1 through P_n. In the following section, we will investigate the possible reaction force sets.

Computation of Possible Reaction Force Sets

In order to obtain force set P_i corresponding to the i-th contact pair using eq.(14), we have to compute a

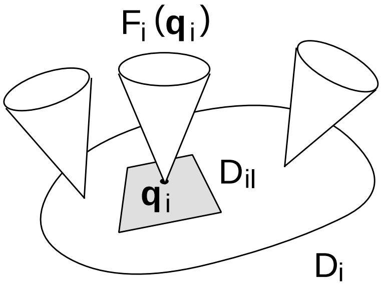


Figure 3. Interpolation of polyhedral convex cones

union of PCC's over a continuous region D_i. In this section, we develop a technique to approximate a union of PCC's from a finite number of configurations instead of the continuous region.

Let us divide the region D_i into a finite number of small regions, D_{i1} through D_{iL}, as shown in Figure 3. Set P_i then consists of unions of PCC's over individual small regions:

$$P_i = \bigcup_{l=1}^{L} \bigcup_{\boldsymbol{q}_i \in D_{il}} F_i(\boldsymbol{q}_i). \tag{17}$$

According to (Hirai and Asada, 90), a union of PCC's over a small region D_{il} can be approximated by a convex sum of PCC's at finite number of points in the region:

$$\bigcup_{\boldsymbol{q}_i \in D_{il}} F_i(\boldsymbol{q}_i) = F_i(\boldsymbol{q}_{i1}) + \cdots + F_i(\boldsymbol{q}_{im}), \tag{18}$$

where $\boldsymbol{q}_{i1}$ through $\boldsymbol{q}_{im}$ are representative configurations in region D_{il}. The union over D_i is then given by the union of the obtained convex sums. Recall that the convex sum of PCC's is also a PCC. Therefore, the possible reaction force set P_i given by eq.(14) is approximated by a union of finite number of PCC's as follows:

$$P_i = \bigcup_{l=1}^{L} P_{il}, \tag{19}$$

where P_{i1} is the convex sum corresponding to the l-th small region. Note that the convex sum of PCC's can be computed on a computer using the computation algorithms of PCC's. Thus, possible reaction force set P_i can be derived on a computer from the model of the workpiece and the fixtures.

Recall that the convex sum of PCC's is also given by a PCC. Substituting eq.(19) into eq.(16) and expanding the obtained equation, we find that set P is described by a union of a finite number of PCC's. In order to investigate whether condition (15) is satisfied or not, let us firstly compute the simplest form of the union of PCC's. The simplest form of the union consists of the smallest number of PCC's. If a union of two PCC's, A and B, coincides to their convex sum $A + B$, union $A \cup B$ can be replaced by the convex sum, which is also a PCC. This implies that two PCC's are reduced into one PCC if the above condition is satisfied. According to (Hirai and Asada, 90), we can examine whether a union of two PCC's coincides to their convex sum or not on a computer. Thus, the simplest form of the union of PCC's can be computed by checking this condition. Next, let us investigate whether condition (15) is satisfied or not. Condition (15) is satisfied and force set P covers the whole space if, and only if, the simplest form of the union consists of unique PCC and the cone covers the whole space of force and moment. It can be checked whether a PCC covers the whole space or not by the computation algorithms of PCC's. As a result, we can examine whether condition (15) is satisfied or not for any fixture layout on a computer.

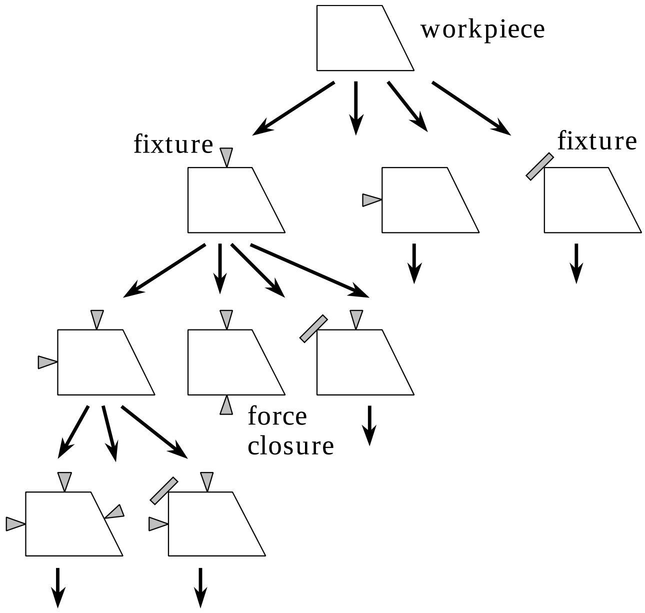


Figure 4. Search tree for fixture layout

Searching Fixture Layout Candidates

As mentioned previous section, eq.(15) provides a necessary condition for force closure clamping. Thus, fixture layout candidates, which have a capability to force closure clamping, can be enumerated by investigating whether individual sets of contact pairs satisfy this equation or not. As shown in Figure 4, let us consider a tree where each node of the tree represent a fixture layout. The root node of the graph denotes a layout where no fixtures are contacting with the workpiece. Since all layouts are described in this tree, fixture layout candidates can be enumerated by searching the tree. Note that all contact pairs involved in node C_n are members of node C_m if node C_n is a descendant of node C_m. Thus, if a layout corresponding to one node satisfies the condition given by eq.(15), its descendents also hold the condition. Namely, it is not necessary to check descendants of the node that satisfies the condition.

Note that possible reaction force set P corresponding to a fixture layout is described by a convex sum of force sets, P_1 through P_n, which denote the range of the reaction forces at individual contact pairs that compose the layout. Thus, if node C_n is a descendant of node C_m, possible reaction force set corresponding to node C_n is given by a convex sum among that of node C_m and the possible reaction force sets at all contact pairs which are involved in node C_n but not involved in node C_m. Namely, we can reduce the computation time of possible reaction force set corresponding to descendant nodes by utilizing that of a parent node.

Considering the above features, let us search a fixture layout tree by breadth-first search starting with the root node of the tree. The computational result of possible reaction force set P at each node is stored so that it can be utilized in the computation of possible reaction force sets at its descendants. If a fixture layout corresponding to a node satisfies the condition given by eq.(15), its descendants are not investigated. Since a set of several fixtures provides a force closure holding in general, it can be expected that the searching procedure does not cause a combinational explosion. Thus, all topological layouts of fixtures that have a capability to force closure holding can be enumerated using this searching procedure.

NUMERICAL EXAMPLE

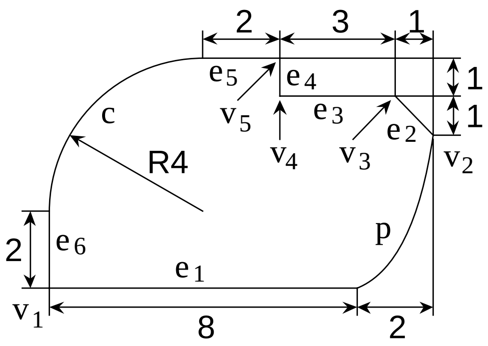
In this section, we will demonstrate the computation process of fixture layout candidates by taking a simple example shown in Figure 5. The planar workpiece shown in Figure 5-(a) consists of the following workpiece elements:

straight edge	e_1 through e_6
circular curve	c
parabolic curve	p
vertex	v_1 through v_5

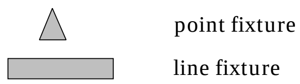
For the sake of simplicity, we assume that point fixture element P and line fixture element L are available, as shown in Figure 5-(b). Straight edges, a circular curve, and a parabolic curve of the workpiece can be held by point fixtures. Vertices v_1, v_2, v_3, and v_5 are convex vertices of the workpiece, which can be clamped by line fixtures. Vertex v_4 is a concave vertex, which can be held by point fixtures. Thus, we have 13 contact pairs as follows:

$$\begin{array}{ll} (e_i, P) & i = 1, 2, \cdots, 6 \\ (c, P) & \\ (p, P) & \\ (v_j, L) & j = 1, 2, 3, 5 \\ (v_4, P) & \end{array}$$

Next, possible reaction force sets are computed for individual contact pairs. Let us demonstrate the computation process of a possible reaction force set by taking the third contact pair, (e_3, P). Let the x-axis and the y-axis in the directions along edges e_1 and e_6, respectively, and vertex v_1 be the origin of the coordinates. Edge e_3 is then described as follows:



(a) Example of workpiece



(b) Fixture elements

Figure 5. Simple example of workpiece and fixture elements

$$\boldsymbol{x}_3(q_3) \triangleq [x_3(q_3), y_3(q_3)] = [9 - q_3, 5]$$

where q_3 is a parameter that specifies the point on edge e_3. The range of parameter q_3 is given by

$$D_3 = \{q_3 \mid 0 \leq q_3 \leq 3\}.$$

Outward normal vector at point $\boldsymbol{x}_3(q_3)$ and tangent vector at that point are given as follows, respectively:

$$\boldsymbol{n}_3(q_3) = [0, 1], \quad \boldsymbol{t}_3(q_3) = [-1, 0]$$

Assume that a point fixture P is in contact with edge e_3 at point $\boldsymbol{x}_3(q_3)$. The set of reaction forces is then described by

$$F_3(q_3) = span\{\boldsymbol{d}_3^n(q_3) \pm \mu \boldsymbol{d}_3^t(q_3)\},$$

where $\boldsymbol{d}_3^n(q_3)$ and $\boldsymbol{d}_3^t(q_3)$ are wrench vectors given by

$$\begin{aligned} \boldsymbol{d}_3^n(q_3) &\triangleq \begin{bmatrix} \boldsymbol{n}_3(q_3) \\ \boldsymbol{x}_3(q_3) \times \boldsymbol{n}_3(q_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 9 - q_3 \end{bmatrix} \\ \boldsymbol{d}_3^t(q_3) &\triangleq \begin{bmatrix} \boldsymbol{t}_3(q_3) \\ \boldsymbol{x}_3(q_3) \times \boldsymbol{t}_3(q_3) \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix} \end{aligned}$$

and μ denotes the coefficient of friction. Dividing the parameter range D_3 into ten small regions, we have the possible reaction force set corresponding to pair (e_3, P) as follows:

$$P_3 = span\{\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3, \boldsymbol{w}_4\},$$

where individual span vectors are given as follows:

$$\begin{aligned} \boldsymbol{w}_1 &= (-0.5, 1.0, 11.5), \\ \boldsymbol{w}_2 &= (0.5, 1.0, 6.5), \\ \boldsymbol{w}_3 &= (-0.5, 1.0, 8.5), \\ \boldsymbol{w}_4 &= (0.5, 1.0, 3.5). \end{aligned}$$

In this computation process, the union of ten PCC's at individual small regions is simplified into sole PCC as shown in the above. Possible reaction force sets corresponding to other contact pairs can be computed in the same way.

Based on the computation of possible reaction force sets, combinations of contact pairs are checked whether the combinations satisfy a necessary condition given by eq.(15). All fixture layout candidates computed using the developed method are listed in Figure 6. In this example, 60 topologically different layouts are found for force closure holding. During the computation process, 466 nodes are checked whether the condition is satisfied or not. Note that the search tree originally consists of $2^{13} = 8192$ nodes. About 94% nodes are not investigated due to the pruning mentioned in the previous section. We have implemented the procedure to enumerate fixture layout candidates on a SUN SPARCstation LX. This procedure was implemented in C and Euslisp (Matsui and Inaba, 90). Possible reaction force sets corresponding to the 13 contact pairs listed before were computed in 2 minutes and 12 seconds. The search tree was investigated in 3 hours and 5 minutes.

CONCLUDING REMARKS

A systematic approach to the generation of fixture layouts has been developed based on the theory of polyhedral convex cones. Firstly, the topological relationship between a workpiece and a set of fixtures was described by listing contact pairs. We found that the static properties of the workpiece strongly depends upon the topological layout of fixtures. Secondly, the static behavior of the workpiece was analyzed and was formulated by use of the cone theory. Force closure condition, which has been proposed as a reliable clamping condition, was also formulated using polyhedral convex cones. Thirdly, a necessary condition for force closure holding was established by introducing possible reaction force sets, which specify the range of reaction forces at individual contact pairs. It was found that we could investigate whether this condition was satisfied or not using the computational algorithms of polyhedral convex cones. Finally, a simple example was shown in order to demonstrate the computational process of fixture layout. We have shown that all possible fixture layouts was derived by use of the proposed method.

Using the developed method, we can enumerate all topological layouts of fixtures from the model of a workpiece and a set of fixtures. This method has a capability of dealing with various workpieces systematically and enables us to plan workpiece holding processes analytically.

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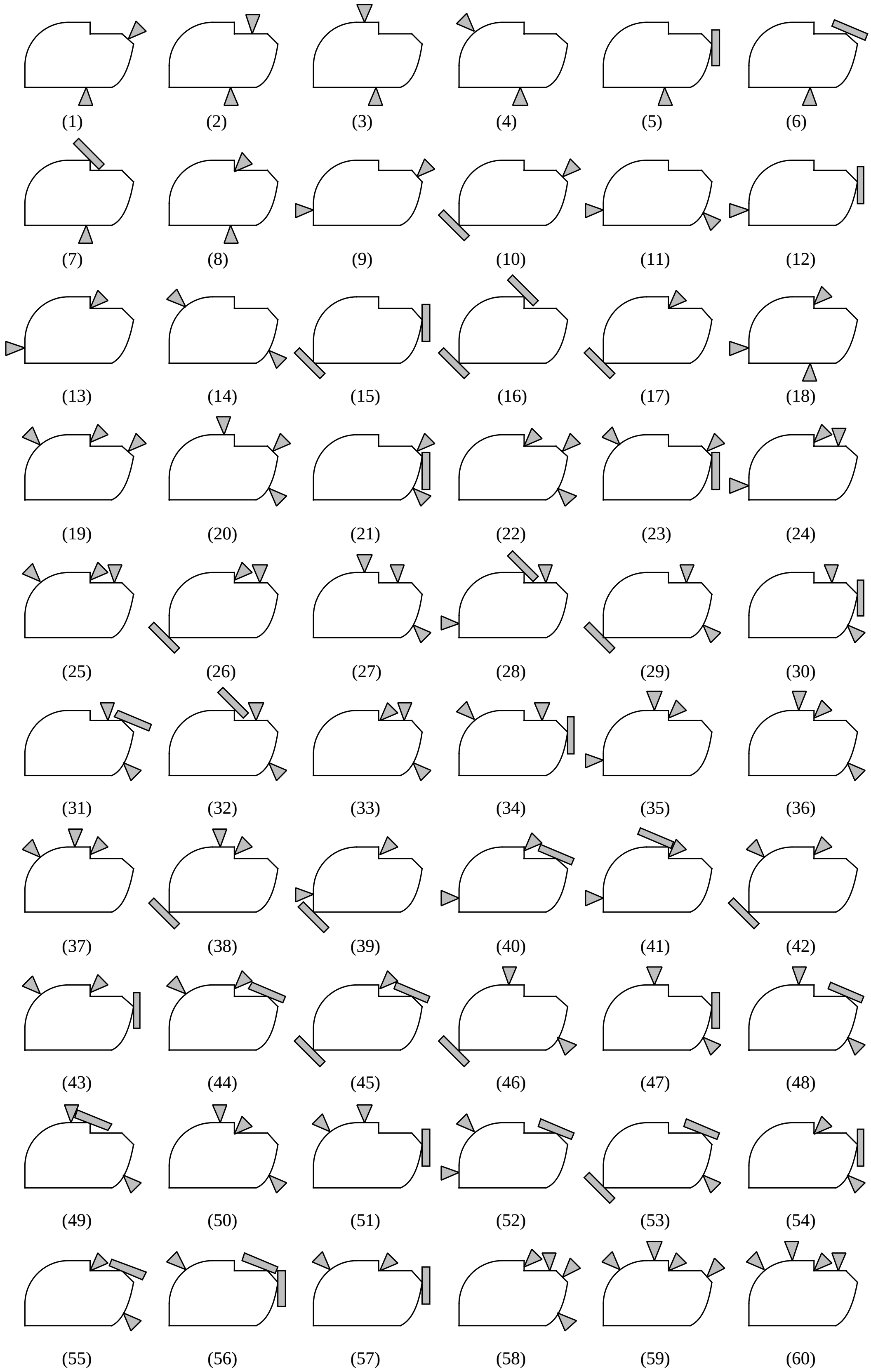

Figure 6. Result of computation of fixture layouts