# Analysis of Hysteresis in Deformation of Rodlike Objects Toward Their Manipulation*

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**A systematic approach to the modeling of deformable rodlike objects is presented. Various rodlike objects such as cords and wires are manipulated in many manufacturing processes. In such processes, it is important for successful manipulation to evaluate their shapes on a computer in advance because their shapes can be changed easily and their deformation often shows hysteresis properties. In this paper, we will develop an analytical method to model the shape of deformable rodlike objects, and using this method, we will analyze their hysteresis properties. First, the potential energy of a rodlike object and the geometric constraints imposed on it are formulated. The shape of the object can be derived by minimizing the potential energy under the geometric constraints. Secondly, procedure to compute the shape of a deformed rodlike object is developed by applying a nonlinear programming technique. Finally, we show some numerical examples and investigate the hysteresis property using our method.**

**Key words: rodlike objects, deformation, hysteresis, statics, manipulation**

## 1. Introduction

Various deformable objects including cords and wires are manipulated in many manufacturing processes. Deformation of these objects is often utilized in order to manipulate them successfully while the manipulation sometimes fails because of unexpected deformation of them. Modeling of deformable objects is thus required so that the shape of the objects can be evaluated on a computer in advance. Especially, to evaluate the shape of rodlike objects is important because their shape can be changed easily by small forces/moments which are imposed on them. There are many studies about the modeling and manipulation of rodlike objects such as flexible beams or wires. Zheng et al derived strategies to insert a flexible beam into a hole without wedging or jamming[1]. We have developed a modeling technique of rodlike objects such as wires considering its static deformation[2]. Nakagaki et al have studied insertion task of a flexible wire into hole using wire model and visual tracking[3]. Wada et al have been analyzed the deformation of knitted fabrics using string model[4].

When we manipulate rodlike objects, the history of the deformation of a rodlike object is important because their shape depends on it. Namely, the shape may change according to the sequence of operations. This hysteresis property must be investigated so that the deformation during a series of operations can be evaluated on a computer.

In this paper, we will develop an analytical method to model the shape of deformable rodlike objects such as cords and wires and we will analyze their hysteresis properties using this method.

First, a geometric representation to describe the shape of a rodlike object with bending and torsional deformation is introduced. The potential energy of the object and the geometric constraints imposed on it are then formulated. The shape of the object in the stable state can be derived by minimizing the potential energy under the geometric constraints. Secondly, procedure to compute the shape of a deformed rodlike object is developed by applying a nonlinear programming technique. Thirdly, numerical examples with the shape transition and the torsional buckling are shown. Finally, the hysteresis property in deformation of a rodlike object is investigated using our method.

## 2. Modeling of Rodlike Object Deformation

### 2.1 Geometric Representation

In this section, we will formulate the geometrical shape of a rodlike object in three-dimensional space. Let $L$ be the length of the object along its central axis and $s$ be the distance from one endpoint of the object along its central axis. In order to describe the deformation of a rodlike object, we will introduce the global space coordinate system and the local object coordinate systems at individual points on the object, as shown in Fig.1. Let $O-xyz$ be the coordinate system fixed on space and $P-\xi\eta\zeta$ be the coordinate system fixed on an arbitrary point $P(s)$ of the object. Select the direction of the local coordinate system $P-\xi\eta\zeta$ so that $\zeta$-axis is aligned with

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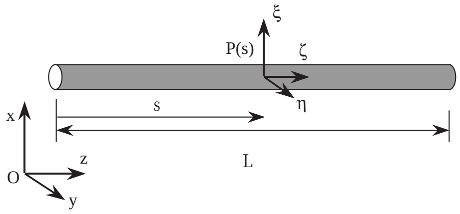
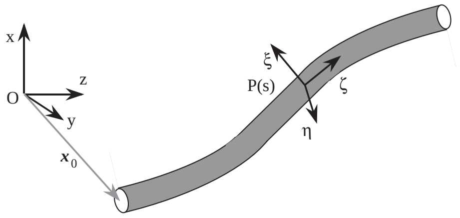



(a) natural shape




(b) deformed shape

Fig. 1 Coordinates systems describing object deformation

the central axis of the object. Let us describe the orientation of the local coordinate system with respect to the space coordinate system by use of Eulerian angles, $\phi(s)$, $\theta(s)$, and $\psi(s)$. In order to express bending and torsional deformation of the object, let us describe the curvature of the object and its torsional ratio. The curvature $\kappa$ and the torsional ratio $\omega$ can be described by use of Eulerian angles $\phi$, $\theta$, and $\psi$ as follows:

$$\begin{aligned} \kappa^2 &= \left(\frac{d\theta}{ds}\right)^2 + \sin^2\theta \left(\frac{d\phi}{ds}\right)^2 \\ \omega^2 &= \left(\frac{d\phi}{ds}\cos\theta + \frac{d\psi}{ds}\right)^2 . \end{aligned} \tag{1}$$

Let $\boldsymbol{x}(s) = [\ x(s),\ y(s),\ z(s)\ ]^{\mathrm{T}}$ be spatial coordinates corresponding to point $P(s)$. The spatial coordinates can be computed as follows:

$$\boldsymbol{x}(s) = \boldsymbol{x}_0 + \int_0^s \begin{bmatrix} \cos\phi\sin\theta \\ \sin\phi\sin\theta \\ \cos\theta \end{bmatrix} ds \tag{2}$$

where $\boldsymbol{x}_0 = [\ x_0,\ y_0,\ z_0\ ]^{\mathrm{T}}$ denotes spatial coordinates at the end point corresponding to $s = 0$.

From the above discussion, we find that the geometrical shape of a deformed rodlike object can be represented by three variables, that is, Eulerian angles $\phi$, $\theta$, and $\psi$. Note that each variable depends upon parameter $s$.

### 2.2 Formulation of Potential Energy and Constraints

Let us formulate the potential energy of a rodlike object. Applying Bernoulli and Navier's assumption, it turns out that the potential energy $U$ is described as follows:

$$U = \frac{1}{2}\int_0^L R_f \kappa^2 ds + \frac{1}{2}\int_0^L R_t \omega^2 ds + \int_0^L D x ds \tag{3}$$

where $R_f$, $R_t$, and $D$ represent the flexural rigidity, the torsional rigidity, and the weight per unit length at point $P(s)$, respectively. Note that $R_f$, $R_t$, and $D$ may vary with respect to variable $s$.

Next, let us represent the geometric constraints imposed on the object. Consider a constraint that specifies the positional relationship between two points on the object. Let $\boldsymbol{l} = [\ l_x,\ l_y,\ l_z\ ]^{\mathrm{T}}$ be a predetermined vector describing the relative position between two operational points, $P(s_a)$ and $P(s_b)$. Recall that the spatial coordinates corresponding to parameter $s$ is given by eq.(2). Thus, the following equational condition must be satisfied:

$$\boldsymbol{x}(s_b) - \boldsymbol{x}(s_a) = \boldsymbol{l} \tag{4}$$

The orientation at some points of the object must be also controlled during the operation. These orientational constraints are simply described as follows:

$$\begin{aligned} \phi(s_c) &= \phi_c \\ \theta(s_c) &= \theta_c \\ \psi(s_c) &= \psi_c \end{aligned} \tag{5}$$

where $\phi_c$, $\theta_c$, and $\psi_c$ are predefined angles at one operational point $P(s_c)$.

Note that any points on a rodlike object must be located outside each obstacle or on it when the contact between the object and rigid obstacles yields. Let us describe the surface of an obstacle fixed on space by function $h(x, y, z) = 0$. Assume that value of the function is positive inside the obstacle and is negative outside it. The condition that a rodlike object is not interfered with this obstacle is then described as follows:

$$h(\boldsymbol{x}(s)) \leq 0, \quad \forall s \in [0, L] \tag{6}$$

where $\boldsymbol{x}(s)$ is described in eq.(2). Note that condition that an object is not interfered with obstacles is described by a set of inequalities, since mechanical contacts between the objects constraints the object motion unidirectionally.

Especially, in order to avoid the interference with itself, a rodlike object must be satisfied the following conditions.

$$\begin{gathered} |\boldsymbol{x}(s_i) - \boldsymbol{x}(s_j)| \geq 2r, \\ \forall s_i, s_j \in [0, L], \ \ \text{s.t.}\ \ |\mathrm{s_i} - \mathrm{s_j}| \geq 2\mathrm{r} \end{gathered} \tag{7}$$

where $r$ represents the radius of a rodlike object.

From the above discussion, the shape of a rodlike object is determined by minimizing potential energy described in eq.(3) under the geometric constraints represented by eqs.(4), (5), (6), and (7). Namely, computation of the shape of a deformed object results in a variational problem under equational and inequality conditions.

## 3. Procedure to Compute Shape of Rodlike Object

Computation of the shape of a rodlike object results in a variational problem as mentioned in the previous section.

In this paper, we will develop a direct method based on Ritz's method [5] and a nonlinear programming technique because the variational problem in this case includes inequality conditions.

Let us express functions $\phi(s)$, $\theta(s)$, and $\psi(s)$ by linear combinations of basic functions $e_1(s)$ through $e_n(s)$:

$$\begin{bmatrix} \phi(s) \\ \theta(s) \\ \psi(s) \end{bmatrix} \triangleq \begin{bmatrix} {\boldsymbol{a}_\phi}^{\mathrm{T}} \\ {\boldsymbol{a}_\theta}^{\mathrm{T}} \\ {\boldsymbol{a}_\psi}^{\mathrm{T}} \end{bmatrix} \cdot \boldsymbol{e}(s) \tag{8}$$

where $\boldsymbol{a}_\phi$, $\boldsymbol{a}_\theta$, and $\boldsymbol{a}_\psi$ are vectors consisting of coefficients corresponding to functions $\phi(s)$, $\theta(s)$, and $\psi(s)$, respectively, and vector $\boldsymbol{e}(s)$ is composed of basic functions $e_1(s)$ through $e_n(s)$. Let us describe the whole coefficient vector $\boldsymbol{a} = [\ \boldsymbol{a}_\phi{}^{\mathrm{T}},\ \boldsymbol{a}_\theta{}^{\mathrm{T}},\ \boldsymbol{a}_\psi{}^{\mathrm{T}}\ ]^{\mathrm{T}}$. Substituting eq.(8) into eq.(3), potential energy $U$ is described by a function of coefficient vector; $\boldsymbol{a}$. The geometric constraints are also described by conditions involving the coefficient vector. As a result, a set of the geometric constraints is expressed by equations and inequalities with respect to the coefficient vectors.

The shape of a deformed rodlike object can be then derived by computing coefficient vector $\boldsymbol{a}$ that minimizes the potential energy under the geometric constraints. This minimization problem under equality and inequality conditions can be solved by use of a nonlinear programming technique such as multiplier method [6]. The shape of the object corresponding to the coefficient vector can be computed by use of eq.(2).

## 4. Numerical Examples

In this section, we will show some numerical examples using our proposed approach. The following set of basic functions $e_1(s)$ through $e_{10}(s)$ are used in the computation of these examples:

$$\begin{aligned} &e_1(s) = 1, \qquad e_2(s) = s, \\ &e_{2n+1}(s) = \sin\frac{n\pi s}{L}, \\ &e_{2n+2}(s) = \cos\frac{n\pi s}{L}. \qquad (n = 1, 2, 3, 4) \end{aligned}$$

The first example shows computed shapes of a rodlike object when it is bent. The second example shows those when it is twisted.

### 4.1 Bending Deformation

The first example shows the topological shape transition of a rodlike object through its bending deformation. Let us align the central axis at both endpoints of a rodlike object in the initial state. We make one endpoint move along this axis in order to shorten the distance between both endpoints. Computed shapes of the object are shown in Fig.2. The shape of the object has one knot as the distance between the endpoints decreases. In this state, the object has not only bending deformation but also has torsional deformation because the potential energy in this state is smaller than that when the object has only bending deformation. Using our proposed approach, we can simulate this shape transition.

### 4.2 Torsional Deformation

The second example shows computational results of a rodlike object in torsional deformation. Let us fix one endpoint of a rodlike object and rotate the other around the central axis. Fig.3 shows computational results. The object becomes curved as the torsional angle becomes larger because the potential energy when the object bends is smaller than that when it keeps itself straight. Thus, we can also simulate this torsional buckling by use of our proposed approach.

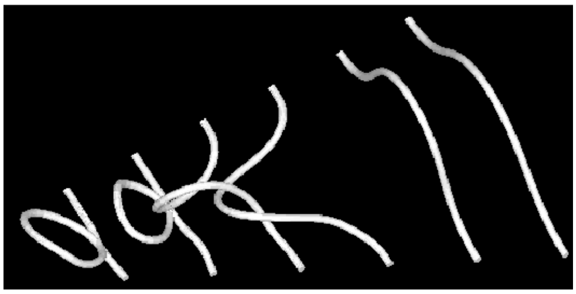


Fig. 2 Computational results in bending deformation

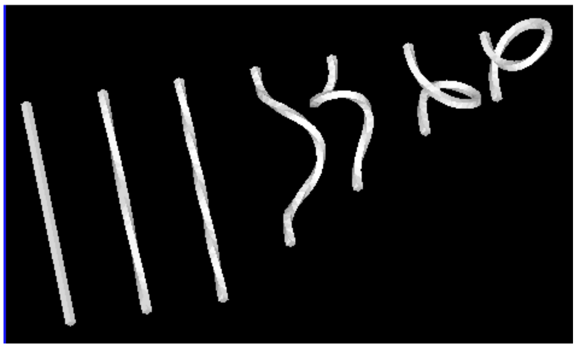


Fig. 3 Computational results in torsional deformation

## 5. Hysteresis Property of Rodlike Object

In this section, we will investigate the hysteresis property of rodlike object deformation.

We experiment with a metal wire, whose flexural and torsional rigidity are $6.6\times10^{-4}$[N$\cdot m^2$] and $2.3\times10^{-4}$[N$\cdot m^2$] respectively, in order to examine that the shape of a rodlike object can depend on the history of manipulative operation. Fig.4 illustrates the devices for this experiment. Two robot hands can control the position and the orientation of both endpoints of a wire. In the initial state, one endpoint is rotated by $\omega_0$[rad] with keeping the wire straight. The distance between two endpoints is then decreased by controlling the two robot hands. We measure the shape of the deformed wire with two cameras. Next, the distance is increased and the shape of the deformed wire is measured.

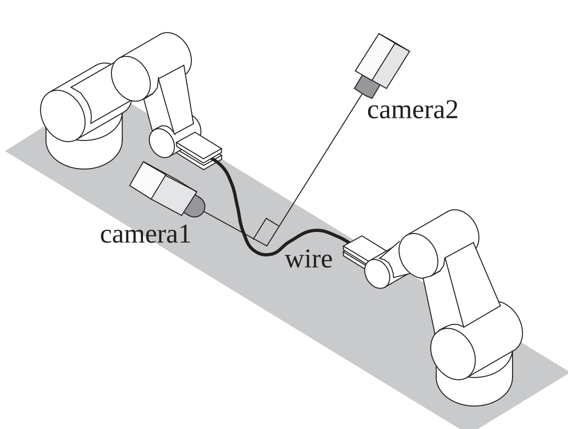
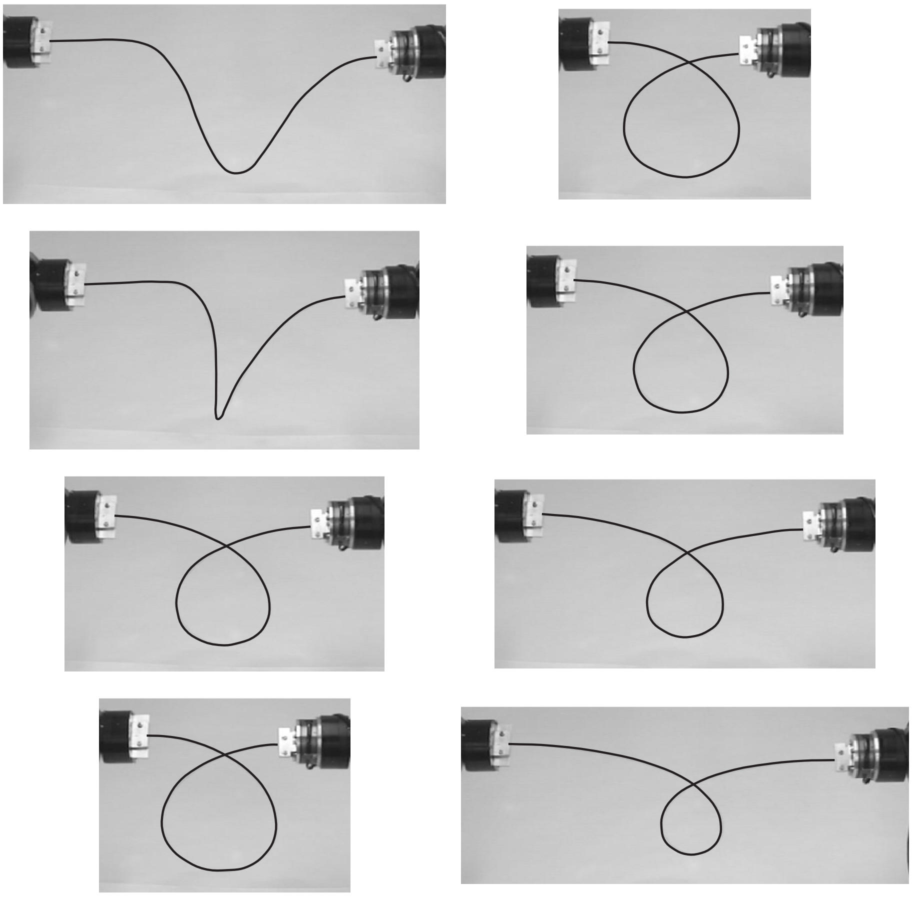



Fig. 4 Sketch of devices for experiment



(a) shortening the distance (b) lengthening the distance

Fig. 5 Experimental results with hysteresis in deformation ( $\omega_0 = 2.25\pi$ )

Fig.5 shows experimental results with hysteresis property. The initial torsional angle $\omega_0$ is equal to $2.25\pi$. When $\omega_0$ is less than $2\pi$, this hysteresis property does not appear. Let us consider this phenomenon.

The shape when a rodlike object is twisted $2\pi$ is equivalent topologically to that when it is bent $2\pi$ as shown in Fig.6. When the shape of the twisted object transits to that of the bent it, the torsional deformation changes to the bending deformation. Because of this shape transition, the potential energy of the object can become smaller than that without the transition. Let us calculate the potential energy of a rodlike object when it is only twisted $2.25\pi$ and that when it is twisted $0.25\pi$ and is bent $2\pi$. Assume that $R_t/R_f$ is equal to the measured value in above experiment, 0.35. Fig.7 shows the computational result. The transverse axis denotes the distance between the endpoints along $z$-axis relative to the object length $L$, that is, $l_z/L$. As the distance between the endpoints is decreased, the potential energy of the object when it is both twisted and bent becomes smaller than that when it is only twisted. Therefore, it seems that the hysteresis property as shown in Fig.5 appears when the potential energy of the object becomes larger than the barrier that divides the type of deformation into two cases as shown in Fig.6. It is difficult to predict when this shape transition occurs but it is important to control it for manipulation of a rodlike object without considering the history of the deformation.

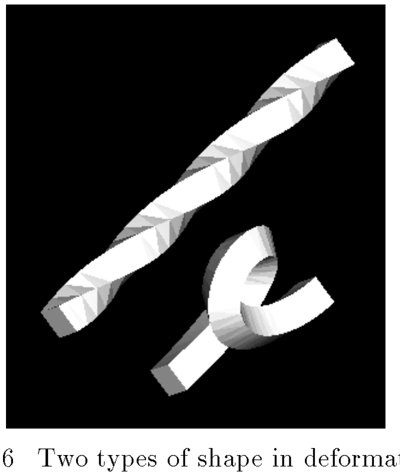


Fig. 6 Two types of shape in deformation

## 6. Conclusion

In this paper, we developed an analytical method to model the shape of deformable rodlike objects and we analyzed their hysteresis properties using our method. First, a geometric representation to describe the shape of a rodlike object was introduced. The potential energy of the object and the

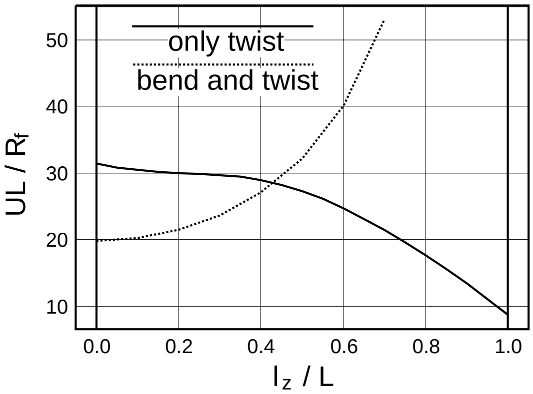



Fig. 7 Comparison of potential energy

geometric constraints imposed on it were formulated using Eulerian angles. The shape of the object in the stable state can be derived by minimizing the potential energy under the geometric constraints. Secondly, procedure to compute the shape of a deformed rodlike object was developed by applying a nonlinear programming technique. Thirdly, numerical examples were shown. we can simulate the shape transition and the torsional buckling. Finally, we investigated the hysteresis property in manipulative operations of rodlike objects using our proposed approach. It is expected that this approach enables us to plan manipulative operations of rodlike objects without hysteresis properties or to utilize such properties for their manipulation.

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