Physical Parameter Identification of Uniform Rheological Deformation Based on FE Simulation
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Abstract
Identification of physical parameters for soft objects is important for surgical simulation, human modeling, and food engineering. In this paper, we propose an approach to estimate the physical parameters of uniform rheological deformation based on 2D/3D finite element (FE) model simulation. At first, the FE dynamic model was described and simulations were done with initial parameters. Then, the identification method was proposed according to analysis of deformation behavior. Finally, identification results were given and validity was evaluated by comparing identified parameters and initial ones. This method can be extended to visco-elastic and layered rheological deformation.

1. Introduction

In surgical training and invasive surgery, precise simulations of human organs and tissues have to be done to describe and predict interaction between deformation and external forces or loads. Such simulation models have been intensively studied since late 80’s and many methods had already been proposed to describe the deformation behavior of soft objects. All of them include important physical parameters which must be available before simulation. Unfortunately, there are little useful data can be obtained to describe these parameters until now.

In recent years, some methods had already been proposed to estimate physical parameters of soft objects. The classification of these methods can be simply described as Fig. 1. First of all, it can be roughly divided into two categories. The first one can be simply described as deformation observation and iterative simulation [1][2]. The second category is theoretical analysis which can be divided into two subcategories. The first one called surface analysis [3][4]and the second one is overall analysis [5][6]. So far, related works in this field mainly focus on elastic or visco-elastic object [7][8]. There are only few papers can be found working on rheological deformation [9][10]. In this paper, we proposed an approach to estimate physical parameters of rheological deformation based on 2D/3D FE simulation. It belongs to the second subcategory according to above classification.

Identification Methods

![Fig.1 Classification of identification methods](image1)

![Fig. 2 3D FE mesh and three element model](image2)
2. Dynamic Model and Simulation

2.1 FE Dynamic Model

Dynamic model of 2D/3D rheological deformation used in this paper can be described by a set of differential equations as follows:

\[
\begin{bmatrix}
1 & -A & -B & -C \\
-A^T & I & -A \omega_\omega & -B \omega_\omega \\
-B^T & -C^T & I & -B \omega_\omega \\
-C^T & -A \omega_\omega & -B \omega_\omega & I
\end{bmatrix} \begin{bmatrix}
\dot{u}_n \\
\dot{v}_n \\
\dot{\lambda}_n \\
\dot{\lambda}_\omega \\
\dot{\Omega}_n \\
\dot{\Omega}_\omega \\
\dot{\Omega}_p
\end{bmatrix} = \begin{bmatrix}
-J_1 \Omega_n - J_\omega \Omega_\omega \\
A^T \left( 2\sigma \nu_n + \sigma \nu_\omega \right) \\
\lambda_\lambda \\
\lambda_\omega \\
\lambda_C \\
\Omega_n \\
\Omega_\omega \\
\Omega_p
\end{bmatrix} - \frac{\lambda_0^2 + \lambda_n^2}{\mu_1^2 + \mu_\omega^2} \begin{bmatrix}
\nu_n \\
B^T \nu_n - d(t) + \nu_\omega \left( B^T \nu_\omega - d(t) \right) \\
C^T \left( 2\sigma \nu_n + \sigma \nu_\omega \right) \\
\lambda_\omega^2 - \lambda_n^2 \nu_n \\
- \frac{\mu_\omega^2}{\mu_1^2 + \mu_\omega^2} \left( \Omega_n - \lambda_n^2 \nu_n \right) \\
- \frac{\mu_\omega^2}{\mu_1^2 + \mu_\omega^2} \left( \Omega_\omega - \lambda_\omega^2 \nu_\omega \right)
\end{bmatrix}
\]

2.2 Simulation of Rheological deformation

In this paper, we perform two simulations with different physical parameters, shown in Table 1. The initial shape of object is found in Fig. 2, in which the length of each element between every two neighboring points is 0.01m. The deformed shapes were shown in Fig. 3. Normal rheological forces and displacements of some nodal points are shown in Fig. 4 and Fig. 5.

Table 1 Simulation parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$E$ (Pa)</th>
<th>$c_1$ (Pa·s)</th>
<th>$c_2$ (Pa·s)</th>
<th>$\gamma$</th>
<th>$d$ (m)</th>
<th>$t_p$ (s)</th>
<th>$t_k$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>8</td>
<td>5</td>
<td>40</td>
<td>0.43</td>
<td>0.006</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Case 2</td>
<td>300</td>
<td>200</td>
<td>500</td>
<td>0.35</td>
<td>0.01</td>
<td>80</td>
<td>60</td>
</tr>
</tbody>
</table>

Fig. 3 Deformed shapes

Fig. 4 Normal rheological forces on bottom surface
3. Identification Method

We can get analytical expression of rheological force in push phase as follows:

$$F_p(t) = F(t_1) e^{\frac{\lambda_1}{\lambda_2} (t - t_1)} + \left(\lambda_2 e^{\mu (t - t_1)} + \mu_2 e^{\mu (t - t_1)}\right) \frac{U^{\text{final}}_{N, t_p}}{t_p} \left(1 - e^{\frac{\lambda_1}{\lambda_2} + \frac{\mu}{\mu_2} (t - t_1)}\right)$$

(1)

The rheological force in keep phase is

$$F_k(t) = F(t_2) e^{\frac{\lambda_1}{\lambda_2} (t - t_2)}$$

(2)

The third equation is

$$F_k(t_3) = \left(\lambda_2 e^{\mu (t - t_3)} + \mu_2 e^{\mu (t - t_3)}\right) u^{\text{final}}_N$$

$$= \frac{\lambda_2 e^{\mu (t - t_3)} + \mu_2 e^{\mu (t - t_3)}}{\lambda_2 e^{\mu (t - t_3)} + \mu_2 e^{\mu (t - t_3)}} F_k(t_3) = \frac{\lambda_2 e^{\mu (t - t_3)} + \mu_2 e^{\mu (t - t_3)}}{\lambda_2 e^{\mu (t - t_3)} + \mu_2 e^{\mu (t - t_3)}} F_k(t_3)$$

(3)

Now, we can estimate analytically physical parameters by solving Eq. 1–3. Identification results can be found in Table 2.

Table 2: Identification results of both cases

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$E$ (Pa)</th>
<th>$c_1$ (Pa·s)</th>
<th>$c_2$ (Pa·s)</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>8.0126</td>
<td>4.9914</td>
<td>40.0069</td>
<td>0.4301</td>
</tr>
<tr>
<td>Error (%)</td>
<td>0.158</td>
<td>0.172</td>
<td>0.017</td>
<td>0.023</td>
</tr>
<tr>
<td>Case 2</td>
<td>300.2574</td>
<td>200.0617</td>
<td>500.3169</td>
<td>0.3502</td>
</tr>
<tr>
<td>Error (%)</td>
<td>0.086</td>
<td>0.031</td>
<td>0.063</td>
<td>0.057</td>
</tr>
</tbody>
</table>

4. Conclusions and Future Works

In this paper, we proposed a method to identify physical parameters of uniform rheological deformation based on 2D/3D FE model simulations. We discussed two deformation behaviors with different parameters and simulation results were given. Then, Identification method was proposed according to the analysis of rheological force and displacement. At last, this method was validated by identification results. Data required in this method include initial and final position of all nodal points.
and normal rheological force on bottom surface. These data can be obtained by using MRI device and force sensor. In addition, this method can be extended to visco-elastic deformation and layered rheological deformation.

In the future, experiments will be performed to validate our method and parameter identification for non-uniform deformation will be done. Then, nonlinear behavior also should be taken into account in rheological deformation.

References


