Two-Dimensional Model of Sliding Motion of a Soft Fingertip Focusing on Stick-to-Slip Transition

○ Van Anh Ho(立命館大学) 平井 慎一 (立命館大学)

Abstract— This paper focuses on analyzing dynamic sliding motion of a hemispherical soft fingertip on a surface. To investigate the deformation of the fingertip during this process, we consider the soft fingertip as if it composed of a finite number of elastic cantilevers which are compressible and bendable. Simulation will be carried out firstly on the 2-dimension (2D) model of soft fingertip, focusing on the analysis of incident slip-page. After that, by using a micro sensor embedded inside the soft fingertip, we will verify by some experimental results.

Key Words: sliding motion, soft fingertip, cantilevers, incident slippage.

1. Introduction

Recently, there has been a large number of robotic researches focusing on manipulation of object by using soft fingered robotic hands. These researches are categorized into two main groups. Firstly, studies focused on analysis of contact mechanics between various soft fingers and the objects [1]. Secondly, human imitated tactile sensing systems have been developed, in according with developments of many kinds of sensors, to emulate human abilities in object grasping and dexterous handling [2]. Whistle the former ones put their center of attention on analyzing stable grasping or object posture controlling by utilizing soft fingertips' compliance during pushing or rolling motion of them on surface of objects; the latter ones concentrated tactile texture perceptions of the sensory fingertips, especially incipient slip detection, during the sliding motions on the objects' surface. Therefore, there has been no research to be a "bridge" between theoretical and experimental research of sliding motion of the soft fingertip. Requirement of a proper theory to explain and verify experimental results is therefore proposed.

In this research, we try to find a theoretical background to explain experimental results of sliding motion of a soft fingertip. Two-dimensional model of a soft fingertip with a finite number of elastic cantilevers is proposed to investigate this sliding motion. Moment before and after the incipient slip which takes an important role in robotic manipulation was focused and compared with experimental results.

2. Model of soft fingertip

Inoue $et\ al.$ [1] proposed a soft fingertip model which comprised of an infinite number of vertical elastic virtual springs to investigate the deformation of the fingertip during pushing or rolling motion on the object. However, this model is not sufficient to model sliding motion with appearance of frictional force. To investigate private motion of each spring on the contact surface, we extended this model by putting interaction between springs, hereafter called cantilevers (Fig. 1). This model maintains the ability of representing the diverse deformations of the soft fingertip during sliding motion in which the soft fingertip is pushed and slid at the same time. These cantilevers are fixed on the equatorial surface of the fingertip, and the free ends are on the outer surface of the fingertip. Each cantilever has the uniform cross sectional area which is circular with radius is dr; whereas length of cantilevers are different with each other depending on their coordinates within the fingertip. Regularly, theoretical researches beforehand eliminated role of frictional force in their dynamic model. However, it is frictional force takes a crucial role in stabilizing objects grasping of robotic hands. By dividing the soft fingertip into many cantilevers in our advanced model of the fingertip, surface-to-surface contact between the fingertip and the objects splits to form point-to-surface contacts between free ends of cantilevers and the objects. Therefore, Coulomb's friction law can be applied to each free end of the cantilever on the contact surface [3]. As a result, by observing dynamic motion of all free ends of the cantilevers the fingertip with appearances of frictional forces, we can assess the transient responses, especially the incipient slip, during sliding motion of the fingertip.

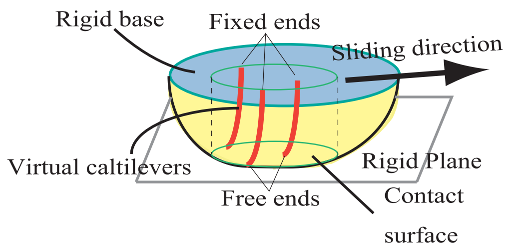

Fig.1 Proposed model of a soft fingertip comprising of virtual elastic cantilevers.

In this paper, we will report the 2-D model of the soft fingertip, as a preliminary step of the research. Let r be soft fingertip's radius. Firstly, let us analyze the distribution of forces inside the soft fingertip when it is pushed perpendicular to the plane with contact depth d_n. In this case, all cantilevers are also pushed vertically with different normal de-formations. At this time, distribution of nor-

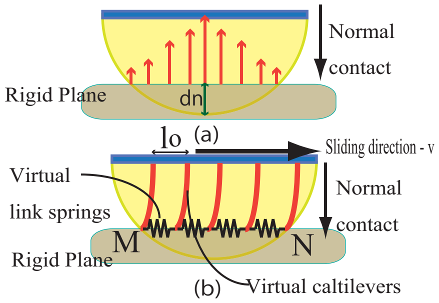

Fig.2 Proposed model of a soft fingertip comprising of virtual elastic cantilevers.

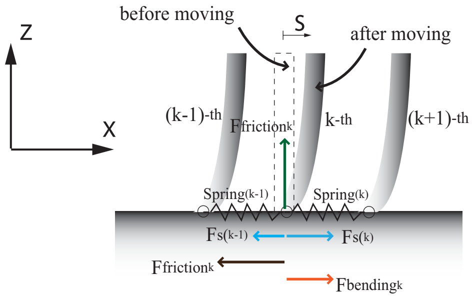

Fig.3 Forces acting on $k-th$ cantilever.

mal forces acting on free ends of cantilevers is plotted in Fig. 2(a) based on the equations originated in [1]. It is evident that highest value of normal force is at the *center* cantilever at which the maximum compression is recorded; while it decreases gradually toward the edge. Therefore, when the fingertip starts to move (but it is not the gross sliding), it is intuitive that regions near the edges will give way and slide short distance before the center starts to slip. This motion around the peripheral zone appears in the form of vibrations, and if the center is still fixed, overall motion does not occur ([4]). Thus, it is necessary to observe dynamic motion of each free end of cantilever on the contact surface.

In order to simplify the complications, three assumptions are given as followed:

1. When the cantilever is bent, its deformation is remarkable only at the free end.
2. Interactions between continuous cantilevers only happen mainly between their free ends on the contact surface.
3. Only cantilevers whose free ends are acting on the contact surface were taken into account to investigate. Cantilevers positioning outside contact surface were minor to the sliding motion of the fingertip.

Basing on above assumptions, we proposed the entire 2-D model of soft fingertip contacting and sliding on a rigid plane. Other than model in Fig. 1, we consider that between two continuous cantilevers' free ends there is a small horizontal virtual spring, hereafter called *link spring* (Fig. 2). These springs represent interaction between cantilevers, and they will take important roles to form vibrations on the outer surface of the fingertip on the contact surface ([4], [5]). Stiffness of each spring is similar and depends on distance between axes of two contiguous cantilever beams, and the cross section as well.

Consequently, by using this model, it is possible to observe motion of each cantilever's free end on the contact surface. By this way, we can assess more detailed transient period between stick and slip states of the fingertip when sliding on the plane surface. Because of the fact that, it is not possible to obtain common motion equations for the entire soft fingertip with appearance of frictional force, discrete method was employed.

3. Simulation

We implemented a simulation of the soft fingertip using the aforementioned model in the transient period between the stick to slip state during the sliding motion of it. In this simulation, the fingertip which has the radius r is first pushed with contact depth d_n. After that, it is started to move with constant velocity v. By observing the vibration of each cantilever's free end, we can perceive the entire motion of the fingertip in this period.

3·1 Parameters Calculation

Firstly, the Young's modulus E of the soft fingertip used in the simulation was measured by conducting a compression test on polyurethane gel and implementing a linear approximation, is 0.2032 MPa [1]. To specify the number of cantilevers, the radius of cross sectional are of the cylindrical cantilever is set to dr. Moreover, this number is obviously dependent on the contact depth d_n as well. Therefore, number n of cantilevers on the contact surface is calculated based on the length MN, which is MN/l_0, in which l_0 equals to distance between two continuous cantilevers' neutral axes (Fig. 2). Let number of cantilevers be $n = 2N + 1$. The *center* cantilever is numbered 0. The cantilevers which distributed on the right side of *center* one are numbered from 1 to N, whereas the left ones are from -1 to -N. Bending stiffness of $k-th$ ($k \in [-N, N]$) cantilever is also calculated based on its parameters as well based on the equation:

$$k_k{}^b = \frac{3EI}{l_k{}^3} = \frac{3E\pi dr^4}{l_k{}^3} \tag{1}$$

with l^k is the natural length of the $k-th$ cantilever. The stiffness of each *link* string is similar and specified by:

$$k_s = \frac{Es_0}{l_0} = \frac{E\pi dr_0^2}{l_0} \tag{2}$$

with dr_0 is radius of cross sectional area of the *link* spring.

3·2 Force/moment analysis

Let detach one random cantilever, called $k-th$ cantilever. We shall see which external forces are acting on it. Fig. 3 shows a detached $k-th$ cantilever with interactions with two neighbored *link* springs $(k-1)-th$, and

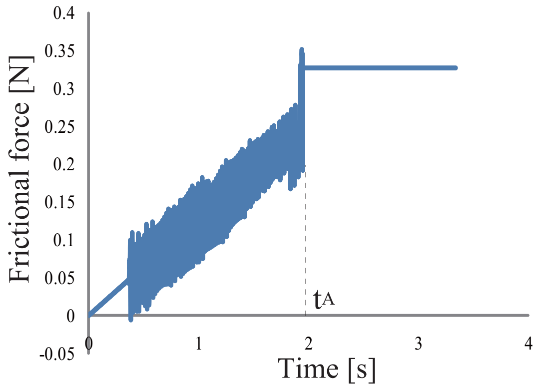

Fig.4 Simulation results: Respose of frictional force acting on the contact surface during stick-to-slip transition.

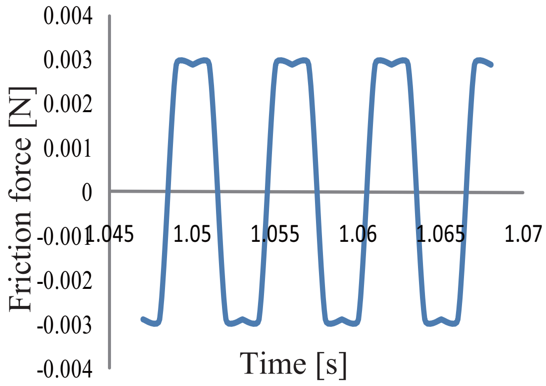

Fig.5 Simulation results: Magnification of response of frictional force acting on a specific cantilever's free end.

$k-th$. The normal force F_k^n is caused by normal deformation of the fingertip, or cantilever as well. The bending force $F_k^{bending} = k_k^b s$ appears when the cantilever is bent. Two elastic forces F_k^s, F_{k-1}^s caused by deformation of $(k-1)-th$ and $k-th$ springs when the free end of the $k-th$ cantilever starts to move. Moreover, there is tangential frictional force acting on the freed end of the cantilever $F_k^{friction}$ (Fig. 3). Therefore, motion equation of the cantilever is formed as followed:

$$F_k^{bending} + F_k^s - F_{k-1}^s - F_k^{friction} = \Gamma_k \qquad (3)$$

During the sticking period, $\Gamma_k = 0$. When $F_k^{friction}$ reaches value of $F_k^n\mu$, Γ_k equals to $m_k\ddot{u_k}$ in which m_k is mass of $k-th$ cantilever, u_k is deviation of the free end after starting to slide. As a result, motion of this cantilever has two stages depending on states of the dynamic frictional force. In the first stage, free end of the cantilever sticks to the surface, and the friction force keeps increasing. When the friction force reaches its maximum defined by: $F_k^n\mu$, the free end starts to slide. After this moment, the frictional force keeps unchanged. For other cantilevers, the acting force/moment are similar, except the $(-N)-th$ cantilever and the $N-th$ cantilever at which there are no $-(N-1)-th\ link$ spring and $(N+1)-th\ link$ spring, respectively. Motion equation for all the cantilevers, i.e. soft fingertip, during sliding motion will be quoted as followed:

$$\mathcal{K}\mathbf{u} = m\ddot{\mathbf{u}} - \sum_{k=-N}^{N} F_k^{friction} + \mathbf{K_b}s \qquad (4)$$

with $s = vt$ is moved distance of fingertip at time t; $\mathbf{u} = [u_{-N}, .., u_k, .., u_N]^t$, $\mathbf{K_b} = [k_{-N}^b, .., k_k^b, .., k_N^b]^t$,

$$\mathcal{K} = \begin{pmatrix} k_{-N}^b - k^s & k^s & & & \\ k^s & k_{-N+1}^b - 2k^s & k^s & & \\ & \vdots & \vdots & \vdots & \\ & & & k_N^b - k^s & k^s \end{pmatrix}$$

During the simulation time, value of frictional force acting on each cantilever's free end is calculated and based on that, stick or slip state of this free end will be decided. The overt slip of soft fingertip on the contact surface will happen when all the free ends slide.

3·3 Simulation results

To conduct the simulation, let us have radius of soft fingertip r = 10mm; contact depth d_n = 2mm; dr=1mm, dr_0=1mm. In this case, N = 6. Fig. 4 shows the response of the total friction force acting over the contact surface during the stick-to-slip period of the fingertip which is moved with velocity v = 2mm/s. It is evident that when the fingertip starts to move with constant velocity, it still sticks to contacting plane causing deformation of it. Therefore, the value of the friction force increases. This value is calculated by sum up all the friction forces acting on cantilevers' free ends. We can observe the fluctuation on the increasing slope of the total friction force. It causes by vibrations of free ends on the contact surface during this period. These vibrations of cantilevers cause the directions of friction forces acting on frees end change frequently, i.e. signs of fiction force values. Let us take a deeper look on the value of friction force acting on one cantilever's free end in Fig. 5. The friction force changes its saturation from positive value to negative one continuously, corresponding to the vibration in position of this free end Fig. h. At the moment of t_A = 2s (Fig. 4), the friction force reaches its highest value, corresponding to the moment at which the last free end started to slide. After this, all the free ends stop vibrating, and move with constant velocity v. Consequently, it is vibration on the contact surface of the soft fingertip really happens and takes a crucial role in transition stick-to-slip period.

As aforementioned, Cutkosky *et al.* [4] suggested that during the stick-to-slip transition, the center of the contact surface was the last one slide right before the gross slippage of the moved fingertip on the object's plane. However, our result is different. Fig. 6 shows the order of first

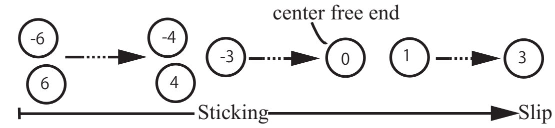

Fig.6 Order of moment of first slip of all the virtual cantilevers' free ends during the stick-to-slip transition.

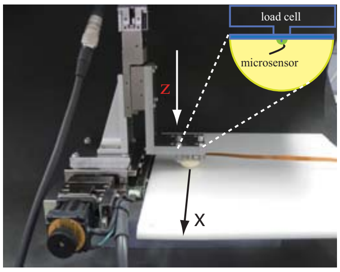

Fig.7 Exeriment set-up.

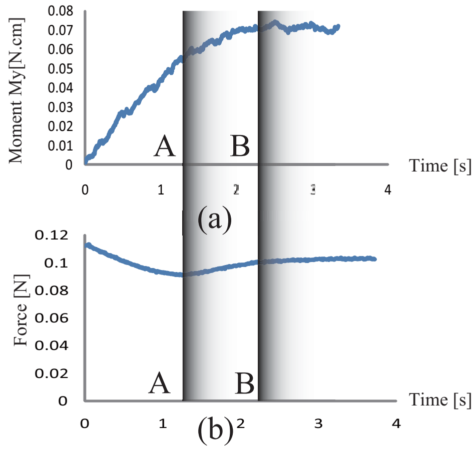

Fig.8 Exerimental results: a) Response of total frictional moment of the load cell. b) Response of normal force from the microsensor on the *center* cantilever.

slide of all cantilevers' free ends on the contact surface before the gross slide of the fingertip. As a result, the last free end started to move is the $3-th$ cantilever. The *center* cantilever's free ends had started to move some moment before that. Thus, even the normal pressure is highest at the center zone of the contact surface; it is not the last one to move before overt slip of the fingertip. This find is considered important for the real application in which incipient slip of the fingertip while contacting with object needs to be detected properly. By using some sensory receptors to detect the sliding of the center area on the contact surface, we can judge the overt slippage of the fingertip timely; because it is usually not fast enough to react to the slippage right after this moment.

4. Experiment

To validate the simulation, we set up an experiment which is illustrated in Fig. 7. A soft fingertip made from polyurethane rubber after an 8-hour curing phase is attached on a 2-DOF (degree of freedom) xz-motorized linear stage to give vertical and horizontal translation of the fingertip on the flat rigid plane. There are two kinds of force/moment sensors employed to measure force/moment acting on the fingertip during the experiment. The 3-DOF micro force/moment sensor is embedded at the center point of the equatorial surface to measure force/moment acting on the fixed end, as well as free end of the *center* cantilever ($0-th$ cantilever) [6]. The 3-DOF load cell PD3 - 32 - 10 - 105 (Nitta, Japan) is fixed upon the fingertip to respond total force/moment acting on the fingertip during sliding motion. Conditions for this experiment are similar to those of simulation. Fig. 8(a) shows the responses of frictional moment acting on the contact surface from the load cell. This graph and graph in Fig. 4 show the similarity in response and the moment at which the gross slippage of the fingertip happens (at point B around 2s). We also observe the fluctuation in the graph of the load cell as well.

Fig. 8(b) show the plot of the normal force F_z of microsensor which measures the normal force acting on the *center* cantilever. According to [6], the moment A when the normal force reaches its minimum value, sliding of *center* zone on the contact surface happens. It is sooner than the moment B when the gross slippage of the contact surface takes place. As a result, this experimental results support simulation results which were reported in Section 3.

5. Conclusions

This paper has just discussed about our initial approach in modeling the dynamic sliding motion of a soft fingertip. Experimental results show remarkably correctness of the model. In the future, 3-dimensional model and multiple states of contact of the soft fingertip will be conducted to fulfill the purpose of research.

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