Contour and Shape Modeling of Wet Material Objects Using Periodic and Closed Smoothing Spline Surfaces

Hiroyuki Fujioka and Hiroyuki Kano

Division of Sciences
Tokyo Denki University, JAPAN

Workshop on Modeling, Identification, and Control of Deformable Soft Objects in IROS2007
Sheraton Hotel and Marina, San Diego, CA, USA, Nov. 2, 2007

Our Projects (1/3)

As is well-known, spline functions have been used in various fields.

e.g. CG, Numerical Analysis, Image Processing, Robotics, etc.

Advantages of using Spline functions

- **Computational feasibilities** (C. de Boor, *A Practical Guide to Splines*, 1978)

  Using B-splines as basis functions yields very simple algorithms for designing curves and surfaces.

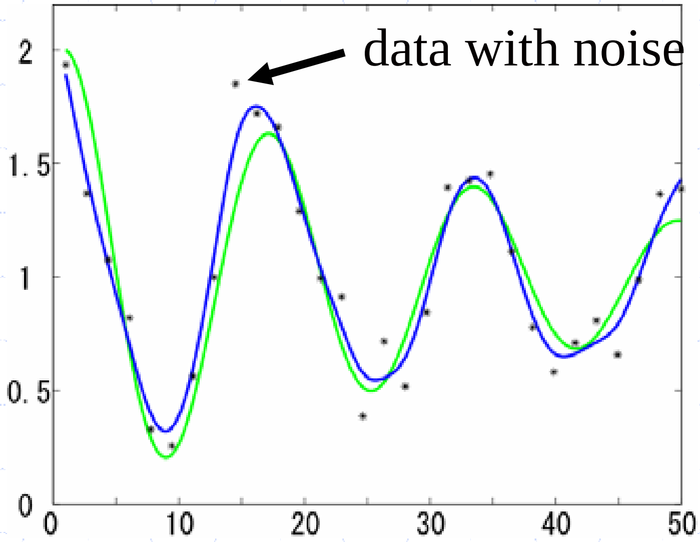
- **Dimensional extendability** (Fujioka, Kano, Egerstedt and Martin, IJICIC 2005)

  The approach using B-splines enables us to extend the results for one-dimensional case to two dimensional case and to even higher dimensions.

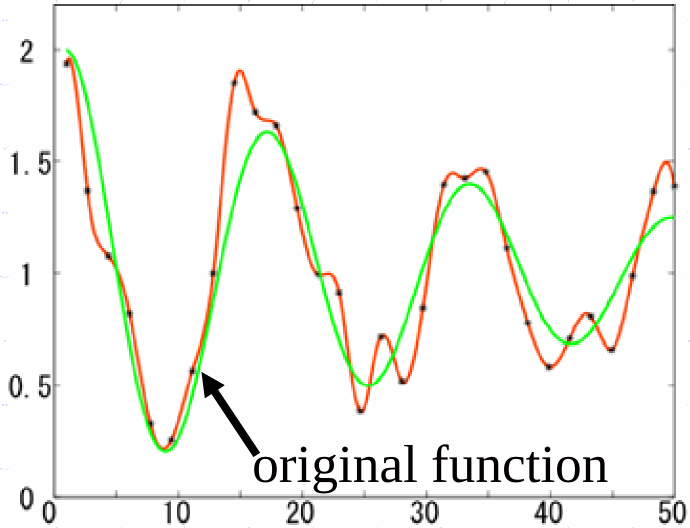
Our Projects (2/3)

Smoothing splines vs Interpolating splines

- Smoothing splines are expected to yield more feasible solutions than interpolating splines in the case where we are given a set of data with noises.



Smoothing splines



Interpolating splines

Properties of Smoothing Splines (Fujioka and Kano et al., 2005)

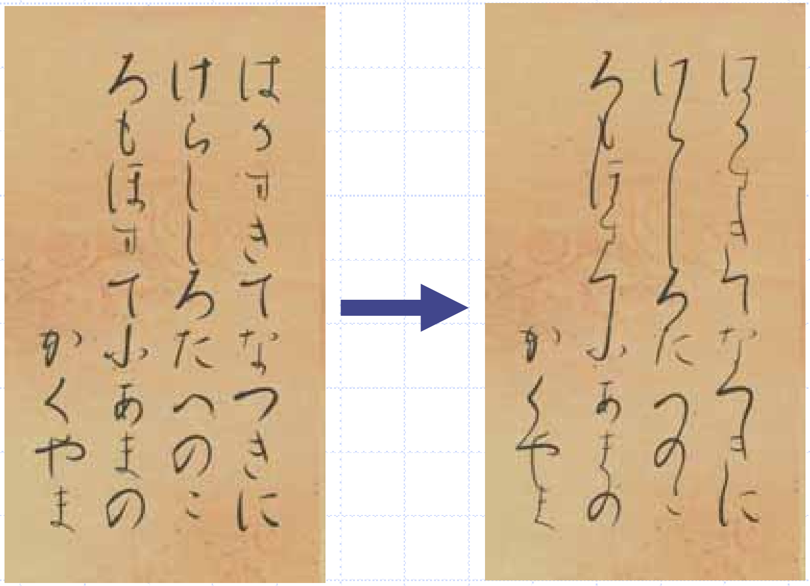
- Using B-spline approaches, we analyzed asymptotic properties of designed spline curves and surfaces when the number of given data increases.

Our Projects (3/3)

An example of Application : Constructing Cursive Characters

(Fujioka and Kano, IEEE Trans SMC '05; AR '06; KAIST IJARM, to appear)

- By using B-splines and the theory of optimal smoothing splines, a scheme is developed for generating and manipulating characters.

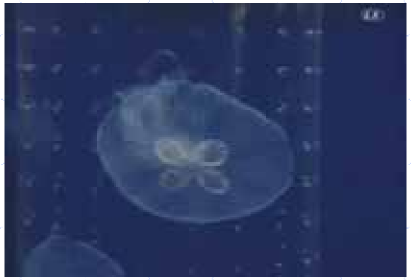


Design example of cursive characters

Constructing characters from human handwriting motion with esthetic evaluation.

1. Introduction (1/2)

Contour and Shape Modeling of Wet Material Objects



- A motion with various deformations is a trademark for wet material objects – such as red blood cell, jellyfish.
- One of important issues is to analyze and understand the motions from the observational data.

The contour and shape modeling of objects may play the key role.

Related study

- The problem of modeling the contour and shape of deformable objects has been studied in the field of image processing.

  e.g. Snake and Active Contour Model.

- The most approaches have focused their attention on the problem of modeling the contour of objects at some time instants.

⇨ **Schemes of modeling the contour and shape are required.**

1. Introduction (2/2)

Main Purpose

We develop a synthesizing scheme for modeling the contour and shape of wet material objects based on the design method of optimal periodic and closed smoothing surfaces.

Outline

- We present basic results for **optimal smoothing spline surfaces**.
- We analyze **statistical properties** of optimal smoothing spline surfaces when the number of data becomes infinity.
- We extend the design and analysis method to the case of **periodic** and **closed splines**.
- The results are applied to model the contour and shape of wet material objects.

2. Optimal Smoothing Spline Surfaces (1/4)

- **Spline Surface**

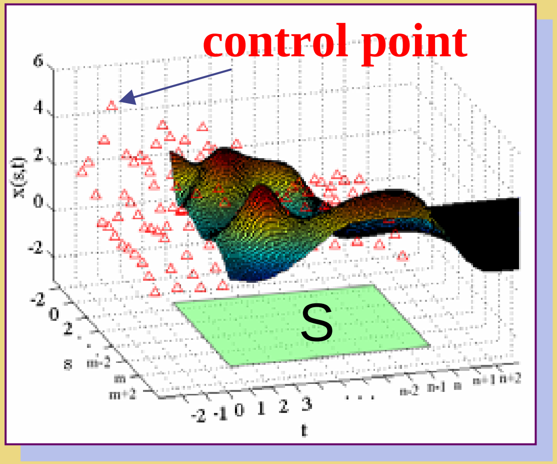
$$x(s,t) = \sum_{i=-k}^{m_1-1} \sum_{j=-k}^{m_2-1} \tau_{i,j} B_k(\alpha(s - s_i)) B_k(\beta(t - t_j))$$

$B_k(\cdot)$: normalized uniform B-splines with degree k $(k = 3)$.

α, β : positive constants.

$\tau_{i,j}$: weighting coefficient called **control point**.

$m_1, m_2 (> 2)$: integers.



Choosing appropriate $\tau_{i,j}$, $x(s,t)$ can represent arbitrary spline surface on the rectangular domain

$$S = [s_0, s_{m_1}] \times [t_0, t_{m_2}].$$

2. Optimal Smoothing Spline Surfaces (2/4)

Smoothing Splines

Suppose that we are given a set of data

$$D = \{ (u_i, v_i, d_i) \; : \; (u_i, v_i) \in S, \; d_i \in R; \; i = 1, 2, \cdots, N \}.$$

Let $\alpha = [\alpha_{i,j}] \in R^{M_1 \times M_2}$ be the weight matrix with $M_1 = m_1 + 3, M_2 = m_2 + 3.$

Problem 1

Find a $\alpha \in R^{M_1 \times M_2}$ minimizing the cost function

$$J(\alpha) = \lambda \int_{s_0}^{s_{m_1}} \int_{t_0}^{t_{m_2}} \{ \nabla^2 x(s,t) \}^2 ds dt + \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} w_{ij} \left(x(u_i, v_j) - d_{ij} \right)^2 .$$

$\lambda (> 0)$: smoothing parameter,

$w_{ij} (0 \le w_{ij} \le 1)$: weights for approximation errors.

2. Optimal Smoothing Spline Surfaces (3/4)

Optimal Solution

Optimal weight ω is obtained as a solution of

$$\left(\lambda Q + BWB^T\right)\hat{\omega} = BWd,$$

where

$$\hat{\omega} = \mathrm{vec}\,\omega \in \mathbb{R}^{M_1 M_2}; \quad B = B_2 \otimes B_1$$

$$B_1 = [b_1(u_1)\ \ b_1(u_2)\ \ \cdots\ \ b_1(u_{N_1})] \in \mathbb{R}^{M_1 \times N_1}$$

$$B_2 = [b_2(v_1)\ \ b_2(v_2)\ \ \cdots\ \ b_2(v_{N_2})] \in \mathbb{R}^{M_2 \times N_2}$$

$$W = \mathrm{diag}[w_{11}\ \ w_{21}\ \ \cdots\ \ w_{N_1 N_2}] \in \mathbb{R}^{N_1 N_2 \times N_1 N_2}$$

$$d = [d_{11}\ \ d_{21}\ \ \cdots\ \ d_{N_1 N_2}]^T \in \mathbb{R}^{N_1 N_2}$$

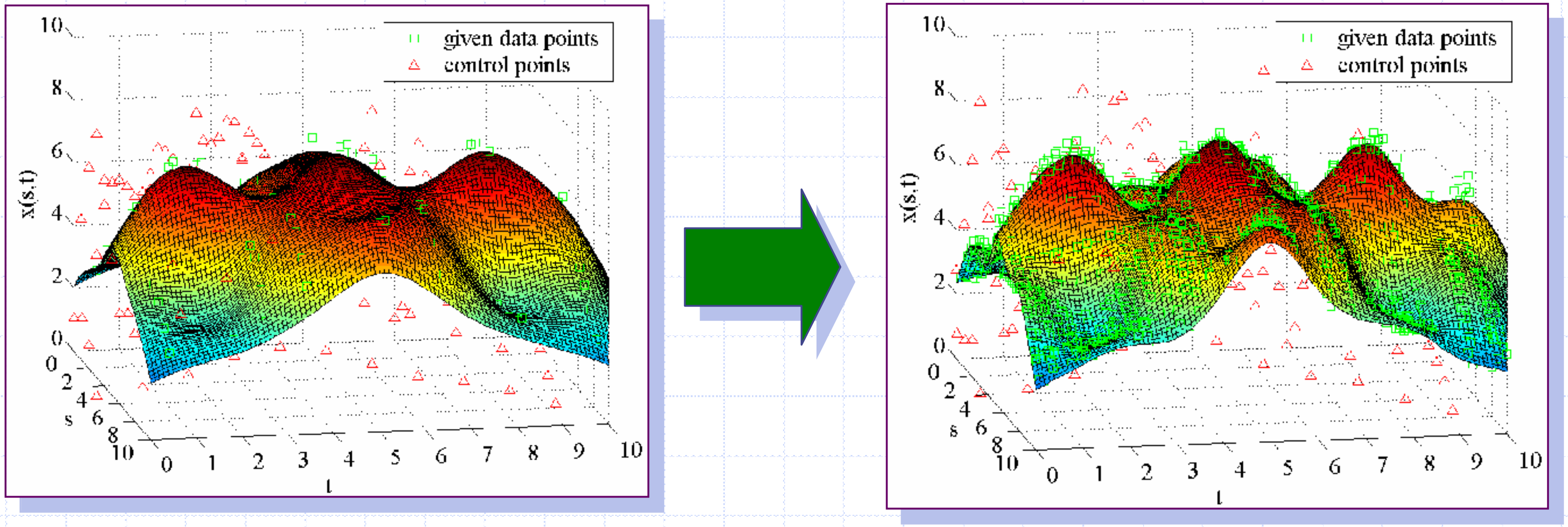
$$b_l = [B_3(\alpha(s - s_{-3}))\ \ B_3(\alpha(s - s_{-2}))\ \ \cdots\ \ B_3(\alpha(s - s_{m_l - 1}))]^T \in \mathbb{R}^{N_1 N_2}$$

$$Q = Q_2^{(00)} \otimes Q_1^{(22)} + Q_2^{(02)} \otimes \left(Q_1^{(02)}\right)^T + \left(Q_2^{(02)}\right)^T \otimes Q_1^{(02)} + Q_2^{(22)} \otimes Q_1^{(00)};$$

$$Q_l^{(ij)} = \int_{I_l} \frac{d^i b_l(t)}{dt^i} \frac{d^j b_l^T(t)}{dt^j}\, dt.$$

3. Properties of Smoothing Splines for Sampled Data (1/2)

We analyze asymptotic and statistical properties by assuming that the data is obtained by sampling some surfaces $f(s;t)$ with or without noises.



■ Main Result **(Fujioka, Kano, Egerstedt and Martin, 2005)**

Under some natural condition, the optimal smoothing spline surfaces converge to some limiting surfaces as the number of sampled data increases.

3. Properties of Smoothing Splines for Sampled Data (2/2)

We can show that the control points θ of smoothing surfaces designed for a set of sampled data, converge to control points θ_c of limiting surfaces.

Theorem 1

Assume that integration intervals I_1, I_2 *are* $I_1 = (s_0, s_{m_1}),\ \ I_2 = (t_0, t_{m_2})$ *and that the condition (A1) holds. Then*

- θ_{N_1, N_2} *converges to* θ_c *as* $N_1, N_2 \to \infty$.
- $E\{\hat{\theta}_{N_1,N_2}\} = \theta_{N_1,N_2}$, *and* $\hat{\theta}_{N_1,N_2}$ *converges to* θ_c *as* $N_1, N_2 \to \infty$ *in the mean square sense.*

(A1) The sample points $(u_i, v_j),\ i = 1, \ldots, N_1;\ j = 1, \ldots, N_2$, are such that

$$\lim_{N_1, N_2 \to \infty} \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} g(u_i, v_j) = \int_{s_0}^{s_{m_1}} \int_{t_0}^{t_{m_2}} g(s, t)\, ds\, dt$$

for every continuous function $g(s, t)$ in $[s_0, s_{m_1}] \times [t_0, t_{m_2}]$.

4. Periodic and Closed Smoothing Spline Surfaces (1/3)

Problem 2 (Periodic and Closed Smoothing splines)

Find a $\alpha \in \mathbb{R}^{M_1 \times M_2}$ minimizing the cost function

$$J(\alpha) = \lambda \int_{s_0}^{s_{m_1}} \int_{t_0}^{t_{m_2}} \left\{ \nabla^2 x(s,t) \right\}^2 ds\,dt + \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} w_{ij} \left(x(u_i, v_j) - d_{ij} \right)^2 .$$

subject to the **continuity constraints**

$$\frac{\partial^l}{\partial s^l} x(s_0, t) = \frac{\partial^l}{\partial s^l} x(s_{m_1}, t), \quad \forall t \in [t_0, t_{m_2}], \quad l = 0, 1, 2,$$

or/and

$$\frac{\partial^l}{\partial t^l} x(s, t_0) = \frac{\partial^l}{\partial t^l} x(s, t_{m_2}), \quad \forall s \in [s_0, s_{m_1}], \quad l = 0, 1, 2.$$

- Note: **Periodic case** ･･･ periodic in either s or t

  Closed case ･･･ periodic in both s and t

4. Periodic and Closed Smoothing Spline Surfaces (2/3)

Problem 2 (Periodic smoothing splines)

Find a $\tau \in \mathbb{R}^{M_1 \times M_2}$ minimizing the cost function

$$J(\tau) = \lambda \int_{s_0}^{s_{m_1}} \int_{t_0}^{t_{m_2}} \left(\nabla^2 x(s,t) \right)^2 ds\,dt + \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} w_{ij} \left(x(u_i, v_j) - d_{ij} \right)^2 .$$

subject to the **continuity constraints**

$$\frac{\partial^l}{\partial t^l} x(s, t_0) = \frac{\partial^l}{\partial t^l} x(s, t_{m_2}), \quad \forall s \in [s_0, s_{m_1}], \quad l = 0, 1, 2.$$

- Using B-splines, the constraint can be written as a **linear constraint**

$$G\boldsymbol{\tau} = 0,$$

where $G = [I_{3M_1 \times 3M_1} \;\; 0_{3M_1 \times (M_1 M_2 - 6M_1)} \;\; -I_{3M_1 \times 3M_1}] \in \mathbb{R}^{M_1 \times M_1 M_2}.$

Minimizing the cost function subject to the constraint $G\boldsymbol{\tau} = 0$, is now a straightforward task.

4. Periodic and Closed Smoothing Spline Surfaces (3/3)

Optimal Solution

Optimal weight $\Theta \in R^{M_1 \times M_2}$ is obtained as a solution $\theta = \mathrm{vec}\,\Theta \in R^{M_1 M_2}$ of

$$\begin{bmatrix} \eta Q + BWB^T & G^T \\ G & 0_{3M_1 \times 3M_1} \end{bmatrix} \begin{bmatrix} \theta \\ \frac{1}{2}\omega \end{bmatrix} = \begin{bmatrix} BWd \\ 0_{3M_1} \end{bmatrix}.$$

Note

- This equation is consistent,

$$\mathrm{rank} \begin{bmatrix} \eta Q + BWB^T & G^T & BWD \\ G & 0_{3M_1 \times 3M_1} & 0_{3M_1} \end{bmatrix} = \mathrm{rank} \begin{bmatrix} \eta Q + BWB^T & G^T \\ G & 0_{3M_1 \times 3M_1} \end{bmatrix}.$$

- If $\eta Q + BWB^T > 0$, then the coefficient matrix is nonsingular, and the solution exists uniquely.
- The same assertion for asymptotic and statistical properties holds.

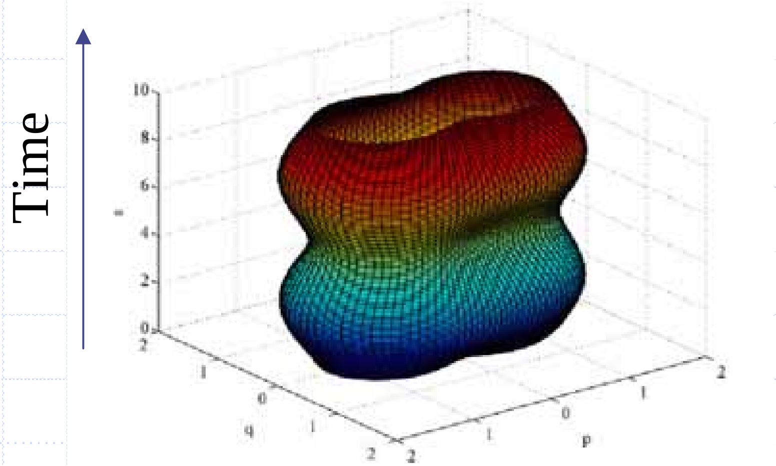
5. Contour and Shape Modeling (1/6)

Aim

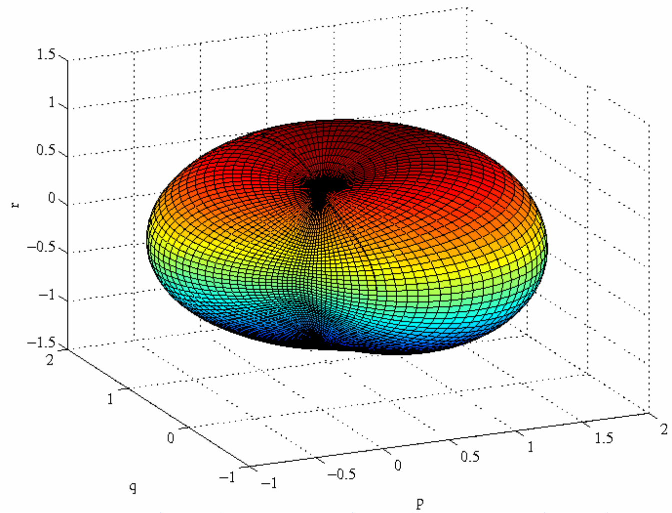
- We apply the method for designing optimal periodic and closed smoothing splines to the problem of contour and shape modeling of wet material objects.
- The effectiveness is examined by numerical and **experimental** studies.

Numerical Study (Contour and Shape modeling of RBC)

- The purpose is to verify the convergence properties when the number of data increases to the infinity.



Contour model

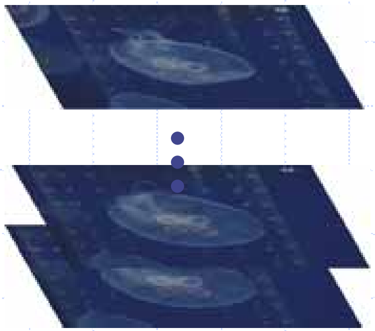

Shape model

5. Contour and Shape Modeling (2/6)

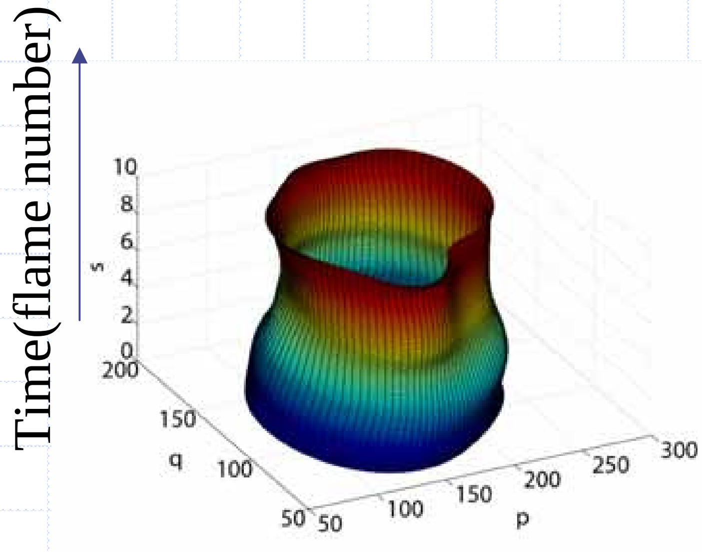
Experimental Study (Dynamic Contour Modeling of Jellyfish)

We consider the problem of modeling the jellyfish motion with deformation and translation by using a small number of image frames (11 frames) in movie file (101 frames) .

Movie file (101 frames)



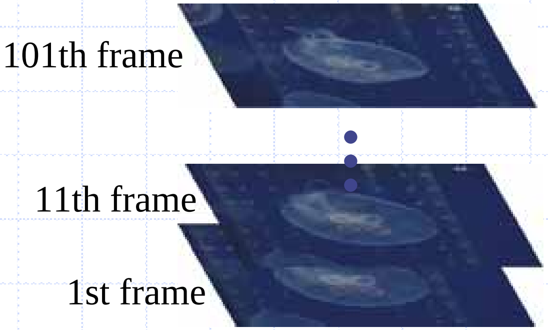
11 frames



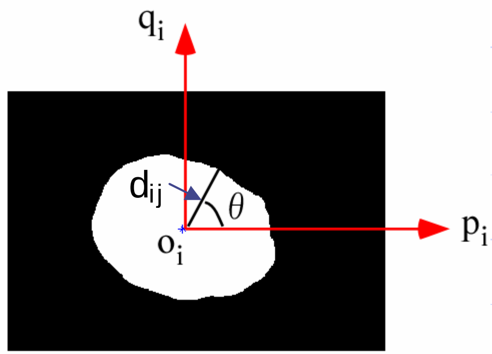
Dynamic Contour Model

5. Contour and Shape Modeling (3/6)

Movie file with 101 [frame]



11 frames



signature

- Modeling Procedure

1. We assume that the i-th frame corresponds to the time $s = 0.1 \times (i - 1)$, and we use only 11 frames obtained by sampling at every 10-th frames starting with 1-st frame.

2. For each i-th frame, we fix $o_i - p_i q_i$ plane with the origin at the centroid of jellyfish.

3. By employing '**signature**', we compute the distance d_{ij} from the centroid to the boundary pixel at each angle $\theta = 0.2\pi v_j$ [rad] with $v_j = j - 1; \ j = 1, 2, \ldots, 10.$

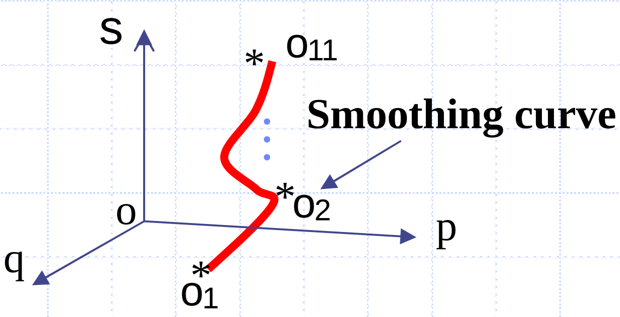
4. We get two sets of data $(u_i; o_i)$ and $(u_i; v_j; d_{ij})$.

5. Contour and Shape Modeling (4/6)

Modeling of Translation and Deformation Motion

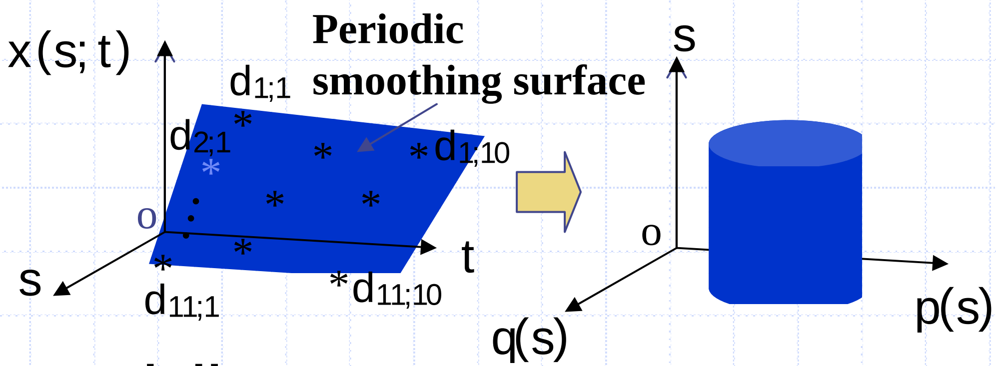
- **Translation**

  For $(u_i; o_i); i = 1; 2; ááá; 11$



- **Deformation**

  For $(u_i; v_j; d_{ij}); i = 1; 2; ááá; 11; j = 1; 2; ááá; 10$



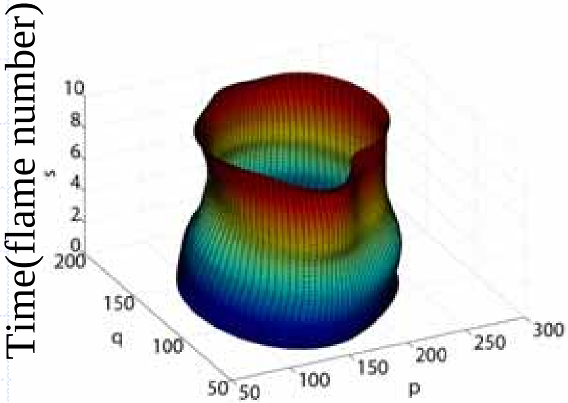
Result of Dynamic Contour Modeling

- **Setup**

$õ = 5â\ 10^{à4}$

$w_i = \frac{1}{N_1 N_2}$

$N_1 = 11; \quad N_2 = 10$



Dynamic contour model of jellyfish.

Movie frame and the corresponding contour from the dynamic model.

5. Contour and Shape Modeling (5/6)

Advantage of Proposed Model

- The model enables us to analyze the motion from various viewpoints.
- For example, the area and the smoothness from the contour model may give meaningful information for evaluating the deformation.

Area:

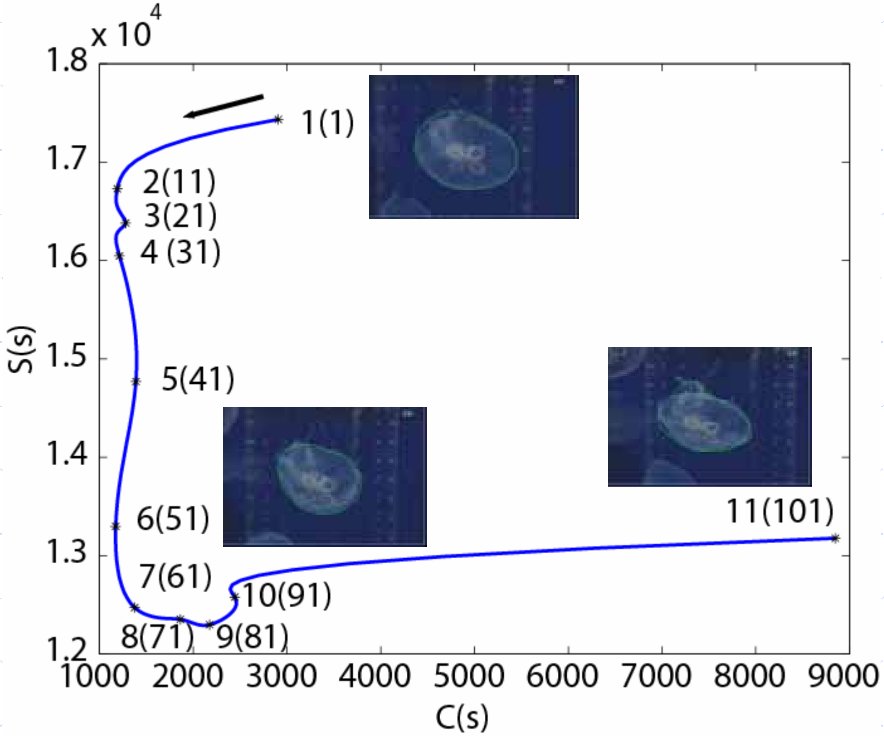
$$S(s) = \frac{1}{t_{m_2}} \left[a^T \left(Q_2^{(00)} \otimes B_c(s) \right) a \right]$$

Smoothness:

$$C(s) = a^T \left(Q_2^{(22)} \otimes B_c(s) \right) a$$

$$B_c(s) = b_1(s) b_1^T(s);$$

$$Q_l^{(ij)} = \int_{I_l} \frac{d^i b_l(t)}{dt^i} \frac{d^j b_l^T(t)}{dt^j} dt;$$



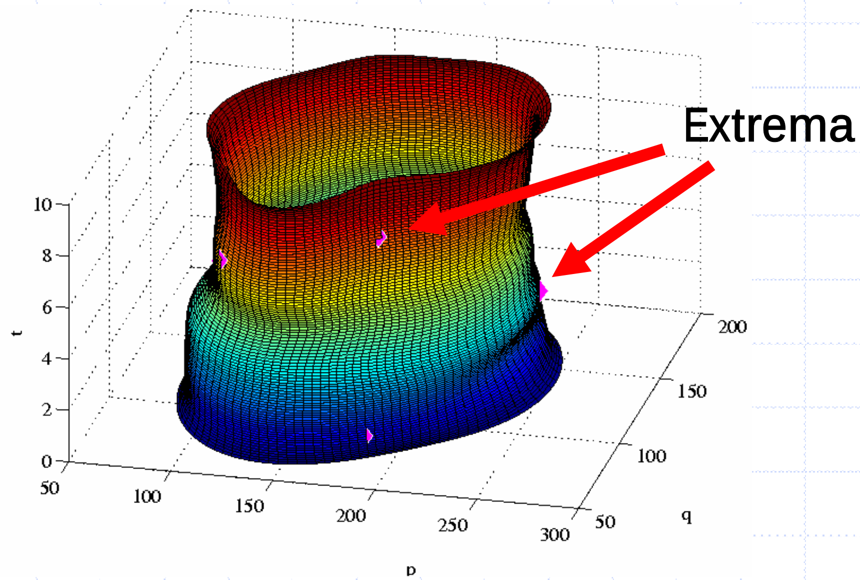
Quantitative evaluation for deformation motion of jellyfish.(parametric representation)

5. Contour and Shape Modeling (6/6)

Advantage of Proposed Model

- The fast computation algorithm of **extrema detection*** may be applied to analyze the deformation motion.

* H. Kano, H. Fujioka and C. Martin, Extrema Detection of Bivariate Spline Functions, *Applied Mathematics and Computation*, Elsevier, to appear.



6. Concluding Remarks

We developed a scheme for modeling the contour and shape of wet material objects based on the design method of optimal periodic smoothing surfaces.

- The basic results for optimal smoothing splines were extended to the periodic and closed case.
- The concise representation of the periodic and closed splines was derived.
- Their statistical and asymptotical properties are given.
- In particular, we applied the theory of periodic smoothing splines to the problem of modeling dynamic contour of wet material objects.

Future Work

- Extending this result to higher dimensional cases, we may construct the 3D dynamic shape model of wet material objects.
- Such studies might be helpful to understand their movements involving deformation.