Analytical Mechanics Report

due date: July 19, 2004
should be submitted to Hirai's room (East 4F)

1. One particle of mass m is free to slide along a smooth cycloidal trough under gravity. The surface of the trough is given by a parametric equations

$$x = \frac{a}{4}(2\theta - \sin 2\theta),$$
$$y = \frac{a}{4}(1 - \cos 2\theta),$$

where a is a positive constant.

(1) Find the Lagrangean of the particle and derive the equation of motion of the particle.

(2) Given mass m, constant a, and initial values $\theta(0)$ and $\dot{\theta}(0)$, plot time histories of the coordinates x and y of the mass and its velocity components $\dot{x}$ and $\dot{y}$. Since the dynamic equation is nonlinear, you may apply numerical computation of ordinary differential equations.

(3) Plot time histories of the potential energy U and the kinetic energy T of the mass.

2. Let us investigate the dynamic 2D bending of an inextensible fiber of length L. Let s be the distance from the left endpoint along the fiber. Let $P(s)$ be a point on the fiber specified by distance s. Let $\theta(s,t)$ be the angle from the horizon along point $P(s)$ at time t. Assume that the fiber is uniform with line density ρ and bend rigidity R. The position of the left endpoint is fixed on the coordinate origin while its orientation is free to rotate. An external torque $\tau(t)$ is applied around the left end point P(0). Bend potential energy U of the fiber at time t is then formulated as

$$U = \int_0^L \frac{1}{2} R \left(\frac{\partial \theta}{\partial s} \right)^2 \, \mathrm{d}s.$$

Kinetic energy T of the fiber at time t is given by

$$T = \int_0^L \frac{1}{2} \rho \left\{ \left(\frac{\partial x}{\partial t} \right)^2 + \left(\frac{\partial y}{\partial t} \right)^2 \right\} \mathrm{d}s$$

where

$$x(s,t) = \int_0^s \cos\theta(u,t) \, \mathrm{d}u,$$
$$y(s,t) = \int_0^s \sin\theta(u,t) \, \mathrm{d}u.$$

Work done by external force is simply formulated as

$$W = \tau\, \theta(0, t).$$

Derive the Lagrange's equation of motion of the fiber that function $\theta(s, t)$ must satisfy. Natural boundary conditions must be described as well.

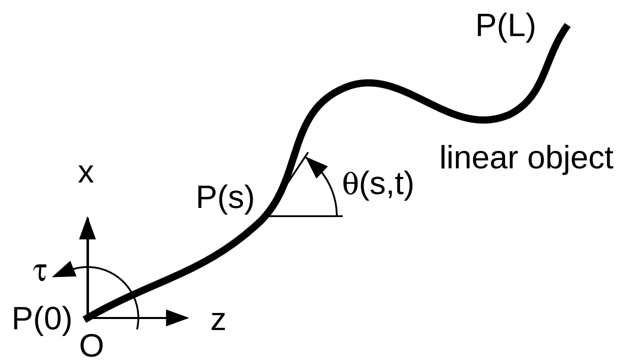

Figure 1: Dynamic 2D bending of inextensible fiber