Constructing Virtual Rheological Objects

Shinichi Tokumoto, Shinichi Hirai, and Hiromi Tanaka†

Dept. of Robotics, Ritsumeikan Univ., Kusatsu, Shiga 525-8577, Japan
†Dept. of Computer Science, Ritsumeikan Univ., Kusatsu, Shiga 525-8577, Japan

E-mail: hirai@se.ritsumei.ac.jp
http://www.ritsumei.ac.jp/se/˜hirai/

Abstract

A new approach to the construction of virtual rheological objects is presented. Construction of virtual rheological objects has not been studied yet though various objects in real world show rheological nature in their deformation. Therefore, a method to construct virtual rheological objects according to actual objects is required to improve the reality in a virtual world.

First, we will select rheological elements appropriate for virtual rheological objects. Next, we will investigate the structures of virtual rheological objects. Then, we will establish a method to identify model parameters through the measuring of actual rheological objects. Finally, we will show the constructed virtual rheological objects.

1 Introduction

In virtual reality, virtual objects should be constructed in a virtual world. Virtual objects must react against actions applied to the objects. It is thus necessary to compute the reactions such as the motion and the deformation of virtual objects according to the actions including forces and displacements applied to the objects. Namely, the construction of virtual objects requires physical models of objects, which determine the relationship between actions and reactions.

Object deformations can be categorized 1) viscoelastic deformation, 2) plastic deformation, and 3) rheological deformation. Construction of virtual viscoelastic objects has been studied extensively [1, 2]. Recently, construction of virtual plastic objects has been investigated [3, 4]. On the other hand, construction of virtual rheological objects has not been studied yet though various objects in real world show rheological nature in their deformation. Rheology has been studied for past several decades and fruitful results have been obtained [5]. Unfortunately, rheology focuses on material properties, say, one-dimensional deformation instead of 2D or 3D deformation of rheological objects.

In this article, we will develop a systematic method to construct virtual rheological objects. First, we will summarize the properties of rheological deformation. Second, we will select rheological elements appropriate for the construction of virtual rheological objects.

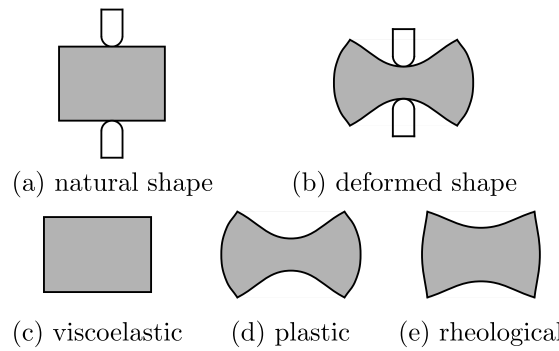


Figure 1: Viscoelastic object, plastic object, and rheological object

Next, we will investigate the structures of virtual rheological objects. It is required to identify parameters in physical models according to actual rheological objects. Thus, we will establish a method to identify model parameters via deformation measuring. We will then show the constructed virtual rheological objects.

2 Rheological Objects

Objects deform according to forces applied to the objects. Objects can be categorized into three with respect to their deformation characteristics. Assume that a natural shape of an object is given as Figure 1-(a). Applying external forces on the object, the object deforms as illustrated in Figure 1-(b). Let us release the applied force and examine the stable shape after the release. Deformation of *viscoelastic objects* is completely lost and their stable shape coincides with their natural shape, as illustrated in Figure 1-(c). Namely, viscoelastic objects have no residual deformation. Deformation of *plastic objects* completely remains and their stable shape coincides with their deformed shape under the applied force, as shown in Figure 1-(d). Namely, plastic objects have no bouncing deformation. Objects with residual deformation and bouncing deformation are referred to as *rheological objects*. Deformation of rheological objects is partially lost after the applied forces are released, as illustrated in Figure

1-(e). Various objects including foods and tissues are categorized into rheological objects.

3 Model Building of Virtual Rheological Objects

3.1 Selection of Rheological Elements

Rheological objects deform according to forces applied to the objects. The relationship between applied forces and object deformation must be described in a physical model. Let us introduce an elastic element and a viscous element so that a physical model can describe the time-dependent deformation of a rheological object. Various deformation properties are then described by the combinations among the two fundamental elements, as listed in Figure 2. These combinations are referred to as *rheological elements*. Then, we have to select rheological elements appropriate for virtual rheological objects.

Recall that deformation properties of a rheological object are summarized as follows:

- residual deformation is involved.
- bouncing displacement is involved.
- vibrations decrease.

Let us examine whether individual rheological elements listed in Figure 2 satisfy the first condition. Let us investigate a rheological element consisting of serially connected two elements. If either of the two has residual deformation, the connected element has residual deformation as well. If both of the two have no residual deformation, the connected element has no residual deformation. These inferences are summarized in Table 1-(a). Let us investigate a rheological element consisting of parallel connected two elements. If both of the two have residual deformation, the connected element has residual deformation. If either of the two has no residual deformation, the connected element has no residual deformation. These inferences are summarized in Table 1-(b). Note that a viscous element has residual deformation while an elastic element has no residual deformation. Thus, we can determine whether a given rheological element has residual deformation or not using Table 1. It has turned out that rheological elements shown in Figure 2-(b), (d), (f), (g), (h), (j), and (k) have residual deformation while the other elements have no residual deformation.

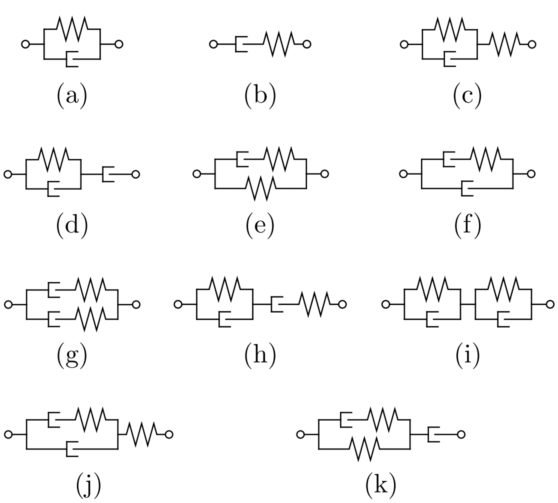


Figure 2: Rheological elements

Table 1: Inferring rules of residual deformation

serial	residual	non-residual
residual	residual	residual
non-residual	residual	non-residual

(a) serial

parallel	residual	non-residual
residual	residual	non-residual
non-residual	non-residual	non-residual

(b) parallel

Table 2: Inferring rules of bouncing deformation

serial	bouncing	non-bouncing
bouncing	bouncing	bouncing
non-bouncing	bouncing	non-bouncing

(a) serial

parallel	bouncing	non-bouncing
bouncing	bouncing	bouncing
non-bouncing	bouncing	non-bouncing

(b) parallel

Let us examine whether individual rheological elements listed in Figure 2 satisfy the second condition. Let us investigate a rheological element consisting of serially connected two elements. If either of the two has bouncing deformation, the connected element has bouncing deformation as well. If both of the two have no bouncing deformation, the connected element has no bouncing deformation. These inferences are summarized in Table 2-(a). Let us investigate a rheological element consisting of parallel connected two elements. If either of the two has bouncing deformation, the connected element has bouncing deformation as well. If both of the two have no bouncing deformation, the connected element has no bouncing deformation. These inferences are summarized in Table 2-(b). Note that an elastic element has bouncing deformation while a viscous element has no bouncing deformation. Thus, we can determine whether a given rheological element has bouncing deformation or not using Table 2. It has turned out that all rheological elements shown in Figure 2 have bouncing deformation.

Let us examine whether individual rheological elements listed in Figure 2 satisfy the third condition. Note that parallel connected elements in a rheological element have the same displacement. Thus, a set of elements which have the same displacement is referred to as a part of a rheological element. One rheological element can be regarded as a series of some parts. If a part involves viscous elements, any vibration on the part converges to zero. On the other hand, vibration on a part consisting of elastic elements alone oscillates and does not converge to zero. Thus, we find that all parts must involve viscous elements to satisfy the third condition. It has turned out that vibration converges to zero for rheological elements shown in Figure 2-(a), (d), (e), (f), (g), (i), and (k) while vibration remains for the other rheological elements.

From the above discussion, we find that rheological elements shown in Figure 2-(d), (f), (g), and (k) satisfy the three conditions. Virtual rheological objects requires simpler elements to improve the response of the objects. Rheological elements in Figure 2-(d) and (f) consist of the least number of fundamental elements. Thus, either of the two is appropriate for constructing virtual rheological objects. In this article, we will use a rheological element shown in Figure 2-(d).

3.2 Formulation of Rheological Elements

Let us formulate a rheological element shown in Figure 2-(d). It has pointed out that linear rheological elements have a difficulty in describing the deformation of actual rheological objects and force-dependent nonlinear dampers should be introduced to tackle this difficulty[6]. Thus, we will introduce a force-dependent damper in the rheological element, as illustrated in Figure 3. This rheological element consists of two parts; the left part is a linear Voigt model and the right part is a force-dependent damper, which has residual deformation. Let P_i and P_j be two end points of the rheological element. Let $\boldsymbol{x}_i = [x_i, y_i, z_i]^T$ be coordinates of point P_i. Length of the Voigt part in the element is denoted as p_{ij} and its natural length is given by p^0_{ij}. Length of the force-dependent damper in the element is then described by $q_{ij} = \| \boldsymbol{x}_i - \boldsymbol{x}_j \| - p_{ij}$. State variables of this rheological element are $\boldsymbol{x}_i$, $\dot{\boldsymbol{x}}_i$, and p_{ij}. Let $\boldsymbol{e}_{ij}$ be a unit vector along $\vec{P_iP_j}$, say, $\boldsymbol{e}_{ij} = (\boldsymbol{x}_i - \boldsymbol{x}_j)/ \| \boldsymbol{x}_i - \boldsymbol{x}_j \|$. Let $\boldsymbol{f}_{ij}$ be a force at point P_i exerted by a rheological element between point P_i and P_j. Note that force $\boldsymbol{f}_{ij}$ coincides with the force caused by the Voigt part, that is,

$$\boldsymbol{f}_{ij} = \{-c_1\dot{p}_{ij} - k_1(p_{ij} - p^0_{ij})\}\boldsymbol{e}_{ij}. \tag{1}$$

Also, force $\boldsymbol{f}_{ij}$ coincides with the force caused by the force-dependent damper, that is,

$$\boldsymbol{f}_{ij} = -c_2\dot{q}_{ij}\boldsymbol{e}_{ij}. \tag{2}$$

Let m_i be mass of point P_i. The equation of motion at point P_i is then described as follows:

$$m_i\ddot{\boldsymbol{x}}_i = \sum_{j\in A_i} \boldsymbol{f}_{ij} + \boldsymbol{f}^{ext}_i, \tag{3}$$

where A_i is a set of points connecting with point P_i and $\boldsymbol{f}^{ext}_i$ is the resultant external force applied to point P_i. Solving eqs.(1), (2), and (3), we can compute the deformation of a rheological object.

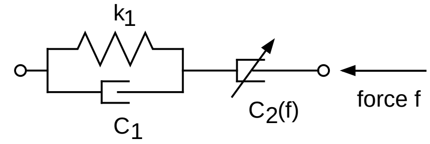


Figure 3: Rheological element with force-dependent damper

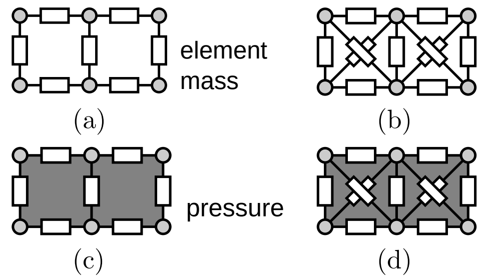


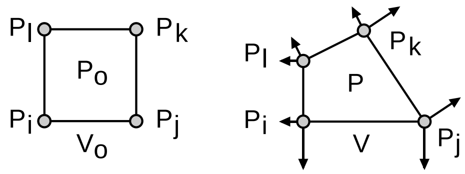
Figure 4: Model structures of virtual rheological object

3.3 Model Structures

In order to describe 2D or 3D deformation of rheological objects, we have to construct 2D or 3D structures using rheological elements. In this section, we will investigate model structures for massive rheological objects.

We will introduce lattice structures to describe 2D or 3D deformation of rheological objects. Lattice structures consist of masses and rheological elements among the masses. One basic structure is illustrated in Figure 4-(a), where rheological elements are arranged vertically and horizontally. Unfortunately, this structure is inappropriate since it cannot retain its own shape. Thus, we have to introduce diagonal elements as illustrated in Figure 4-(b) or have to introduce inside pressure as illustrated in Figure 4-(c). Structure with diagonal elements shows more uniform deformation characteristics than other structures shows. Structure with inside pressure keeps the volume of a rheological object as possible while the structure can change its shape according to external forces. Figure 4-(d) shows a structure with diagonal elements and inside pressure. As mentioned above, individual models have their own properties. Thus, the model structure should be selected through the measurement of 2D or 3D deformation of actual rheological objects.

Let us formulate inside pressure using Figure 5. Let V_0 be the initial volume of a region composed of points P_i, P_j, P_k, and P_l, as illustrated in Figure 5-(a). Let P_0 be the initial pressure inside the region. Note that the inside pressure is in equilibrium with the outside pressure at the initial state. Let V be the volume of the region at a deformed shape and P be the pressure inside the region. Applying Boyle law,



(a) (b)

Figure 5: Inside pressure in region of virtual rheological object

we have $P_0 V_0 = PV$. Namely, inside pressure is given by $P = (P_0 V_0)/V$. Note that $P_0 V_0$ is a predefined constant and V can be computed from coordinates $\boldsymbol{x}_i$, $\boldsymbol{x}_j$, $\boldsymbol{x}_k$, and $\boldsymbol{x}_l$. Thus, we can compute the inside pressure P at individual regions for any deformation of a rheological object. Force $(P - P_0)l_{ij}$ is exerted on a boundary area $P_i P_j$. Let us distribute this force to the end points of the area, say, P_i and P_j, as illustrated in Figure 5-(b). This implies that force $(P - P_0)l_{ij}/2$ outwardly perpendicular to area $P_i P_j$ is exerted on two end points, P_i and P_j. Summing these forces, we can obtain resultant forces caused by inside pressure at individual points.

4 Identification of Physical Properties in Virtual Rheological Objects

In this section, we will identify physical parameters in a rheological element through a creep test on actual rheological objects. Here, we will derive parameters in a rheological element shown in Figure 3. Note that parameters to be identified are k_1 and c_1 in the Voigt part and function $c_2(f)$ in the force-dependent damper.

In a creep test, a constant load is applied to the top face of an object. The height of the object are measured during the test. Note that force in a discrete model is equivalent to stress in an actual continuous object. Thus, we will compute stress applied to an object from the measurements. Let $S(t)$ be area of the top face of the object and $h(t)$ be the height of the object at time t. Let S_0 and h_0 be their initial values, which can be measured in advance. Let W be load applied to the object during the creep test. Assuming that the volume of a rheological object is constant, we have $S(t)h(t) = S_0 h_0$. Stress $P(t)$ is defined as $P(t) = W/S(t)$. These equations yield

$$P(t) = \frac{W}{S_0 h_0} h(t). \tag{4}$$

Note that the right side of the above can be evaluated from measured values. Consequently, we can compute stress applied to an object using eq.(4).

A cubic object of $1.0cm$ in length which consists of a 3:1 mixture of wheat and water is used in a creep test. Constant load of $10gf$, $30gf$, $50gf$, $70gf$, or $90gf$ is applied to the cubic object for 60 seconds before the load is released. Figure 6 shows the measured displacement of an object in a creep test corresponding to load $50gf$. Substituting the measured displacement shown in Figure 6 into eq.(4) yields stress $P(t)$ during the creep test. Computed stress $P(t)$ is plotted in Figure 7.

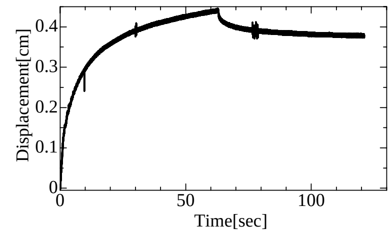


Figure 6: Measured displacement of object

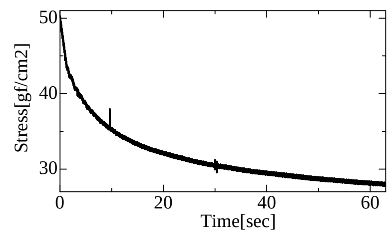


Figure 7: Computed stress applied to object

A rheological element shown in Figure 3 consists of a Voigt part and a force-dependent damper. After the applied load is released, damper coefficient in the force-dependent damper takes a large value since a force applied to the damper is almost equal to zero. This implies that the deformation after the load is released is mainly governed by the Voigt part. Let t_r be the time when the load is released and P_r be stress at this time. Then, bouncing displacement after the load is released is described as follows:

$$x(t) = \frac{P_r}{k_1}\left\{1 - e^{-(k_1/c_1)(t-t_r)}\right\}. \tag{5}$$

From Figure 7, it turns out that stress P_r is equal to $28gf/cm^2$. Figure 8 shows the bouncing displacement after time t_r. Applying least square error method to derive k_1 and c_1, we have found that $k_1 = 470gf/cm^3$ and $c_1 = 2500gfs/cm^3$. Computed bouncing displacement corresponding to these values is plotted by a dotted line in Figure 8.

Next, let us identify a force-dependent damper. It is necessary to compute stress $P(t)$ applied to the

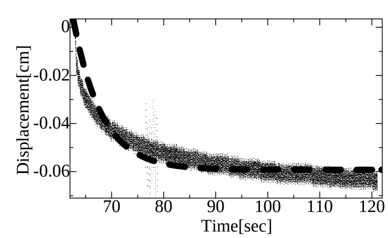


Figure 8: Measured bouncing deformation

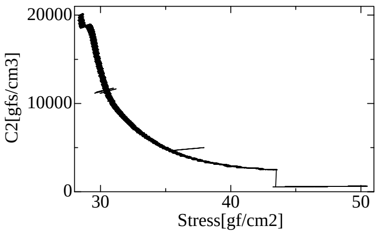


Figure 9: Obtained damper function of force-dependent damper

damper and velocity $v(t)$ of the damper before time t_r to identify a force-dependent damper. Note that displacement of the Voigt part can be computed using the identified values of k_1 and c_1. Subtracting displacement of the Voigt part from the measured displacement plotted in Figure 6 yields the displacement of a force-dependent damper. Velocity $v(t)$ of the damper can be computed by differentiating its displacement. Damper coefficient $c_2(P)$ is then obtained by dividing pressure $P(t)$ by velocity $v(t)$. The obtained $c_2(P)$ is plotted in Figure 9. Function $c_2(P)$ is then assumed as follows:

$$c_2(P) = \begin{cases} \alpha e^{-\beta P_0} & (P \leq P_0) \\ \alpha e^{-\beta P} & (P_0 \leq P \leq P_1) \\ \alpha e^{-\beta P_1} & (P_1 \leq P) \end{cases} \tag{6}$$

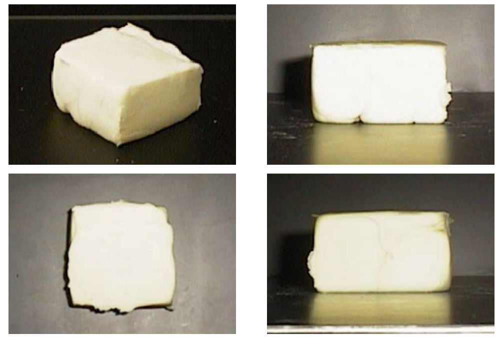
where α, β, P_0, and P_1 are parameters specifying the function. Applying least square error method, we have found that $\alpha = 8.16 \times 10^4 gfs/cm^3$, $\beta = 0.0815$, $P_0 = 9.0 gf/cm^2$, and $P_1 = 80 gf/cm^2$.

5 Simulation and Experimental Evaluation

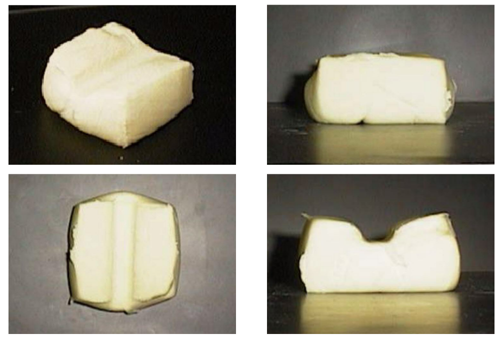
In this section, we will demonstrate virtual rheological objects and will evaluate the object by measuring 3D deformation of actual rheological objects.

Figure 10 shows a measured deformation of a rheological object. The object consists of a 3:1 mixture of wheat and water, which is used in Section 4. The object is $5cm$ in length, $5cm$ in width, and $3cm$ in height. A plate of $1cm$ in width, which is installed on the endpoint of a manipulator, is pressed against the top face of the object. The plate moves downward with velocity $1cm/s$ until it reaches $1cm$ below the top face of the object. Then, the plate remains there for 30 seconds and moves upward until the contact between the plate and the object is released. Figure 10-(a) shows an initial shape of the object and Figure 10-(b) describes a deformed shape in a stationary state. Note that side faces expand at the deformed shape.

Figure 11 shows a computed deformation of a virtual rheological object. This simulation emulates the deformation shown in Figure 10. Physical parameters of the object are identified ones in Section 4. This simulation employs a structure with diagonal elements. Note that side faces expand as the faces of an actual object expand. The expansion of a virtual object is,



(a) initial shape



(b) deformed shape

Figure 10: Deformation of wheat dough

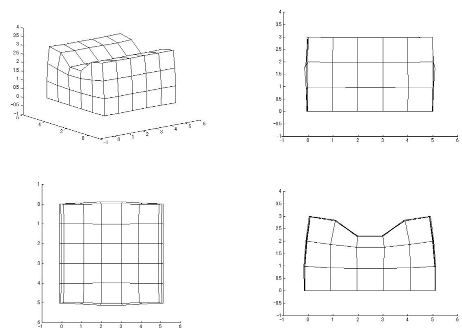
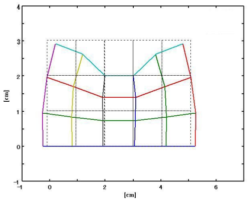


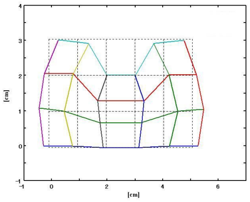
Figure 11: Simulated deformation of wheat dough

however, quite smaller than that of an actual object. This implies that we should select a model structure appropriate for describing the deformation of actual rheological objects.

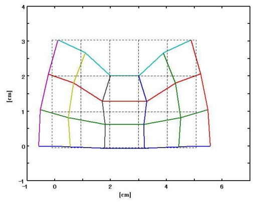
Let us demonstrate the deformation corresponding to different model structures. Figure 12-(a) shows 2D deformation computed by use of a structure with diagonal elements. Figure 12-(b) shows the deformation using a structure with inside pressure. Figure 12-(c) describes the deformation using a structure with diagonal elements and inside pressure. It turns out that expansion in Figure 12-(b) and (c) is larger than that in Figure 12-(a), which is used in the above simulation. Moreover, the shape of expanded side faces in Figure



(a) diagonal elements

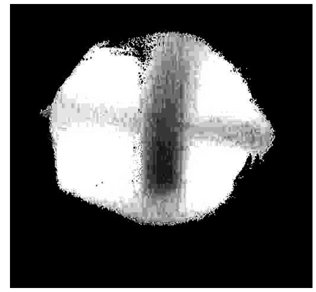


(b) inside pressure

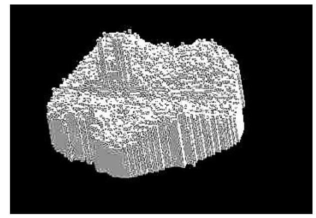


(c) diagonal elements and inside pressure

Figure 12: Simulated deformation corresponding to different model structures



(a) measured distance image



(b) constructed 3D shape

Figure 13: Measured 3D shape of rheological object

12-(b) is more similar to the shape of side faces shown in Figure 10-(b). Consequently, we have concluded that a structure with inside pressure is appropriate for constructing virtual rheological objects.

Identification of physical parameters in a virtual rheological object with inside pressure requires the realtime 3D measuring of object deformation. Shapes during object deformation can be captured using a realtime range finger, CKD CubicScope QSC-100. This range finder can obtain one distance image in 0.3 sec. Shapes during a deformation process are computed from individual distance images. Figure 13-(a) provides a distance image of a deformed rheological object. A cross-shaped rigid object is pressed against the top face of the rheological object. Figure 13-(b) shows a 3D shape constructed from the distance image. As shown in the figure, the range finder can detect the expansion of side faces of a rheological object. As illustrated in Figure 12, expansion of side faces depends on the model structure of a virtual rheological object and physical parameters. This suggests that measuring the expansion enables us to select a model structure and physical parameters appropriate for describing the deformation of an actual rheological object.

6 Concluding Remarks

We have developed a method to construct virtual rheological objects. First, we have selected rheological elements appropriate for describing the deformation of rheological objects and have investigated their model structures. It turns out that three-element model with a force-dependent damper is appropriate. Second, we have established a method to identify model parameters in the element. Then, we have measured the deformation of actual rheological objects to compare them with the deformation of virtual rheological objects. We have found that the deformation of rheological objects can be described but that the expansion of a virtual object differs from that of an actual object. Simulation results have suggested that model with inside pressure should be introduced to virtual rheological objects.

Future issues include 1) constructing virtual rheological objects with inside pressure, 2) identification of their physical parameters, and 3) development of a virtual reality system which can present tactile sensation of rheological objects to humans.

References

[1] Terzopoulos, D., Platt, J., Barr, A., and Fleisher, K., *Elastically Deformable Models*, Computer Graphics, Vol.21, No.4, pp.205–214, 1987

[2] Joukhader, A., Deguet, A., and Laugie, C., *A Collision Model for Rigid and Deformable Bodies*, Proc. IEEE Int. Conf. on Robotics and Automation, pp.982–988, Alberquerque, May, 1998

[3] Galyean, T. A., and Hughes, J. F., *Sculpting: An Interactive Volumetric Modeling Technique*, Computer Graphics, Vol.25, No.4, pp.267–274, 1991

[4] Chai, Y., and Luecke, G. R., *Virtual Clay Modeling Using the ISU Exoskeleton*, Proc. IEEE Virtual Reality Annual Int. Symp., pp.76–80, 1998

[5] Barnes, H. A., Hutton, J. F., and Walters, K., *An Introduction to Rheology*, Elsevier Science Publishers, 1989

[6] Tokumoto, S., Fujita, Y., and Hirai, S., *Deformation Modeling of Viscoelastic Objects for Their Shape Control*, Proc. IEEE Int. Conf. on Robotics and Automation, Vol.1, pp.767–772, Detroit, May, 1999