Deformation Modeling and Path Generation for Linear Object Manipulation

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ABSTRACT

A systematic approach to the deformation modeling of linear objects such as cords or wires and the path generation for their manipulation is developed. First, a deformation modeling method of a linear object is proposed. Its shape can be represented by use of Eulerian angles, and it is computed by minimizing the potential energy under geometric constraints. Secondly, a path generation method for linear object manipulation is proposed. We assume that the path on which the maximum potential energy of the object becomes minimum is most suitable for its manipulation. The optimal path can be derived by introducing the deformation path vector and computing it which minimizes the maximum potential energy during manipulation. Finally, the validity of these methods is demonstrated with a measuring experiment.

Keywords: Linear Objects, Deformation, Modeling, Path Generation, Manipulation

1. INTRODUCTION

Various deformable objects including cords and wires are manipulated in many manufacturing processes. Deformation of these objects is often utilized in order to manipulate them successfully while the manipulation sometimes fails because of unexpected deformation of them. Therefore, modeling of deformable objects is required so that the shape of the objects can be evaluated on a computer in advance. Especially, to evaluate the shape of linear objects is important because their shape can be changed easily by small forces/moments which are imposed on them.

There are many studies about the modeling and manipulation of linear objects such as flexible beams or wires. Zheng et al derived strategies to insert a flexible beam into a hole without wedging or jamming[1]. Nakagaki et al have studied insertion task of a flexible wire into hole using wire model and visual tracking[2]. Wada et al have been analyzed the deformation of knitted fabrics using string model[3]. However, most of proposed methods in such studies are ad hoc and are not applicable to a general manipulation.

In this paper, we will develop a systematic approach to the modeling of linear object deformation and the generation of the optimal path for its manipulation. First, a modeling method of linear object deformation is proposed. The stable shape of the object is computed by minimizing the potential energy under geometric constraints. Some examples are shown in order to demonstrate the effectiveness of this method. Secondly, a method of path generation for linear object manipulation is proposed. The optimal path is derived by minimizing the maximum potential energy of the object during its manipulation. Finally, the validity of these methods is demonstrated by measuring the shape of a vinyl chloride sheet which deforms in two-dimensional space.

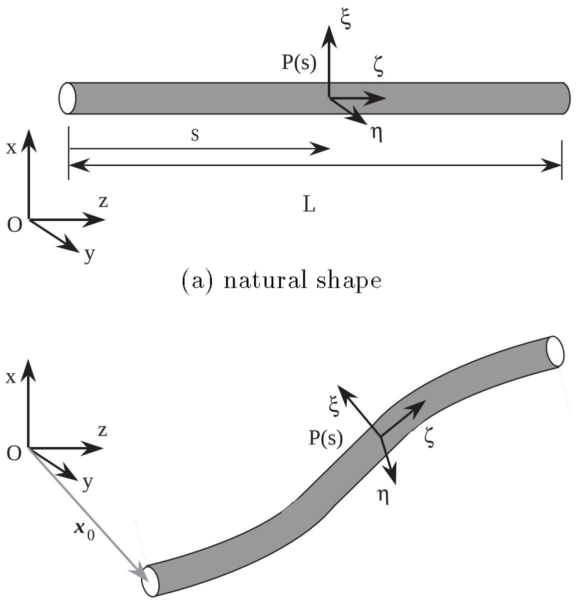
2. MODELING OF LINEAR OBJECT DEFORMATION

Geometric Representation

In this section, we will formulate the geometrical shape of a linear object in three-dimensional space. Let L be the length of a linear object and s be the distance from one endpoint of the object along its central axis. In order to describe the deformation of the object, we will introduce the global space coordinate system and the local object coordinate systems at individual points on the object, as shown in Figure 1. Let $O-xyz$ be the coordinate system fixed on space and $P-\xi\eta\zeta$ be the coordinate system fixed on an arbitrary point $P(s)$ of the object. Select the direction of the local coordinate system $P-\xi\eta\zeta$ so that ζ-axis is aligned with the central axis of the object. Let us describe the direction of the local coordinate system with respect to the space coordinate system by use of Eulerian angles, $\phi(s)$, $\theta(s)$, and $\psi(s)$.

In order to represent bending and torsional deformation of the object, let us describe the curvature of the object and its torsional ratio. The curvature κ and the torsional ratio ω can be described by use of Eulerian angles ϕ, θ, and ψ as follows:

$$\begin{aligned}\kappa^2 &= \left(\frac{d\theta}{ds}\right)^2 + \sin^2\theta\left(\frac{d\phi}{ds}\right)^2 \\ \omega^2 &= \left(\frac{d\phi}{ds}\cos\theta + \frac{d\psi}{ds}\right)^2 .\end{aligned} \tag{1}$$



(a) natural shape

(b) deformed shape

Figure 1: Coordinates systems describing object deformation

Let $\boldsymbol{x}(s) = [\ x(s),\ y(s),\ z(s)\ ]^{\mathrm{T}}$ be spatial coordinates corresponding to point $P(s)$. The spatial coordinates can be computed as follows:

$$\boldsymbol{x}(s) = \boldsymbol{x}_0 + \int_0^s \begin{bmatrix} \cos\phi\sin\theta \\ \sin\phi\sin\theta \\ \cos\theta \end{bmatrix} ds \tag{2}$$

where $\boldsymbol{x}_0 = [\ x_0,\ y_0,\ z_0\ ]^{\mathrm{T}}$ denotes spatial coordinates at the end point corresponding to $s = 0$.

From the above discussion, we find that the geometrical shape of a deformed linear object can be represented by three variables, that is, Eulerian angles ϕ, θ, and ψ. Note that each variable depends upon parameter s.

Formulation of Potential Energy and Constraints

Let us formulate the potential energy of a linear object. Applying Bernoulli and Navier's assumption, it turns out that the potential energy U is described as follows:

$$U = \frac{1}{2}\int_0^L R_f\kappa^2 ds + \frac{1}{2}\int_0^L R_t\omega^2 ds + \int_0^L Dx ds \tag{3}$$

where R_f, R_t, and D represent the flexural rigidity, the torsional rigidity, and the weight per unit length at point $P(s)$, respectively. Note that R_f, R_t, and D may vary with respect to parameter s.

Next, let us represent geometric constraints imposed on the object. Consider a constraint that specifies the positional relationship between two points on the object. Let $\boldsymbol{l} = [\ l_x,\ l_y,\ l_z\ ]^{\mathrm{T}}$ be a predetermined vector describing the relative position between two operational points, $P(s_a)$ and $P(s_b)$. Recall that the spatial coordinates corresponding to parameter s is given by Eq.(2). Thus, the following equational condition must be satisfied:

$$\boldsymbol{x}(s_b) - \boldsymbol{x}(s_a) = \boldsymbol{l} \tag{4}$$

The direction at some points of the object must be also controlled during the operation. These directional constraints are simply described as follows:

$$\begin{aligned} \phi(s_c) &= \phi_c \\ \theta(s_c) &= \theta_c \\ \psi(s_c) &= \psi_c \end{aligned} \tag{5}$$

where ϕ_c, θ_c, and ψ_c are predefined angles at one operational point $P(s_c)$.

Note that any points on a linear object must be located outside each obstacle or on it when the object contacts with some rigid obstacles. Let us describe the surface of an obstacle fixed on space by function $h(x, y, z) = 0$. Assume that value of the function is positive inside the obstacle and is negative outside it. The condition that a linear object is not interfered with this obstacle is then described as follows:

$$h(\boldsymbol{x}(s)) \leq 0, \quad \forall s \in [0, L] \tag{6}$$

where $\boldsymbol{x}(s)$ is described in Eq.(2). Note that condition that an object is not interfered with obstacles is described by a set of inequalities, since mechanical contacts between the objects constraints the object motion unidirectionally.

Especially, in order to avoid the interference with itself, a linear object must be satisfied the following conditions

$$\begin{gathered} |\boldsymbol{x}(s_i) - \boldsymbol{x}(s_j)| \geq d, \\ \forall s_i, s_j \in [0, L],\ \text{s.t.}\ \ |s_i - s_j| \geq d \end{gathered} \tag{7}$$

where d represents the diameter of the object.

From the above discussion, the shape of a linear object is determined by minimizing the potential energy described in Eq.(3) under the geometric constraints represented by Eqs.(4), (5), (6), and (7). Namely, shape computation of deformed linear objects results in a variational problem under equational and inequality conditions.

Procedure to Compute Shape of Linear Object

As mentioned above, the shape of a linear object can be derived by solving a variational problem.

In this paper, we will develop a direct method based on Ritz's method [4] and a nonlinear programming technique because the variational problem in this case includes inequality conditions.

Let us represent functions $\phi(s)$, $\theta(s)$, and $\psi(s)$ by linear combinations of basic functions $e_1(s)$ through $e_n(s)$:

$$\begin{bmatrix} \phi(s) \\ \theta(s) \\ \psi(s) \end{bmatrix} \triangleq \begin{bmatrix} \boldsymbol{a}_\phi{}^{\mathrm{T}} \\ \boldsymbol{a}_\theta{}^{\mathrm{T}} \\ \boldsymbol{a}_\psi{}^{\mathrm{T}} \end{bmatrix} \cdot \boldsymbol{e}(s) \tag{8}$$

where $\boldsymbol{a}_\phi$, $\boldsymbol{a}_\theta$, and $\boldsymbol{a}_\psi$ are vectors consisting of coefficients corresponding to functions $\phi(s)$, $\theta(s)$, and $\psi(s)$, respectively, and vector $\boldsymbol{e}(s)$ is composed of basic functions $e_1(s)$ through $e_n(s)$. Let us describe the whole coefficient vector $\boldsymbol{a}$ as follows:

$$\boldsymbol{a} = [\ \boldsymbol{a}_\phi{}^{\mathrm{T}},\ \boldsymbol{a}_\theta{}^{\mathrm{T}},\ \boldsymbol{a}_\psi{}^{\mathrm{T}}\ ]^{\mathrm{T}}. \tag{9}$$

Substituting Eq.(8) into Eq.(3), potential energy U is described by a function of coefficient vector; $\boldsymbol{a}$. The geometric constraints are also described by conditions involving the coefficient vector. As a result, a set of the geometric constraints is represented by equations and inequalities with respect to the coefficient vector.

The shape of a deformed linear object can be then derived by computing coefficient vector $\boldsymbol{a}$ that minimizes the potential energy under the geometric constraints. This minimization problem under equality and inequality conditions can be solved by use of a nonlinear programming technique such as multiplier method[5]. The shape of the object corresponding to the coefficient vector can be computed by use of Eq.(2).

Numerical Examples of Deformation Modeling

In this section, we will show some numerical examples using our proposed approach. The following set of basic functions $e_1(s)$ through $e_{10}(s)$ are used in the computation of these examples:

$$
\begin{aligned}
&e_1(s) = 1, \quad e_2(s) = s, \\
&e_{2n+1}(s) = \sin \frac{n\pi s}{L}, \\
&e_{2n+2}(s) = \cos \frac{n\pi s}{L}. \quad (n = 1, 2, 3, 4)
\end{aligned}
$$

The first example shows computed shapes of a linear object when it is bent. The second example shows those when it is twisted.

Bending Deformation: The first example shows the topological shape transition of a linear object through its bending deformation. Let us align the central axis at both endpoints of a linear object in the initial state. We make one endpoint move along this axis in order to shorten the distance between both endpoints. Computed shapes of the object are shown in Figure 2. The shape of the object has one knot as the distance between the endpoints decreases. In this state, the object has not only bending deformation but also has torsional deformation because the potential energy in this state is smaller than that when the object has only bending deformation. Using our proposed approach, we can simulate this shape transition.

Torsional Deformation: The second example shows computational results of a linear object in torsional deformation. Let us fix one endpoint of a linear object and rotate the other around the central axis. Figure 3 shows computational results. The object becomes curved as the torsional angle becomes larger because the potential energy when the object bends is smaller than that when it keeps itself straight. Thus, we can also simulate this torsional buckling by use of our proposed approach.

Figure 2: Computational results in bending deformation

Figure 3: Computational results in torsional deformation

3. PATH GENERATION FOR LINEAR OBJECT MANIPULATION

Procedure to Generate Optimal Path

In this paper, we will propose a method to generate the path of hands for manipulation of linear objects by use of the method proposed in the previous section.

The objective of deformable object manipulation is to move them without damage or to configure them so that we can utilize their elastic deformation in general. Therefore, it is not preferable that they deform plastically during their manipulation. Furthermore, excessive potential energy of deformable objects can be easily transformed into kinetic energy by small disturbance. Namely, their shape may become unstable and change dynamically because of excessive static deformation. It is difficult to predict when such dynamic deformation occurs. Therefore, in order to avoid plastic or dynamic deformation, we assume that the path on which the maximum potential energy of a linear object becomes minimum is most suitable for their manipulation.

We can represent the shape transition of a linear object while it deforms from one shape to another shape by use of the coefficient vector which is described as follows:

$$\boldsymbol{a} = (1-k)\boldsymbol{a}_0 + k\boldsymbol{a}_1 + k(1-k)\boldsymbol{b} \quad (0 \le k \le 1) \quad (10)$$

where $\boldsymbol{a}_0$ and $\boldsymbol{a}_1$ represent coefficient vectors corresponding to the shape in the initial state and that in the final state, respectively. k is defined as the state parameter. Note that vector $\boldsymbol{a}$ is equal to vector $\boldsymbol{a}_0$ when $k = 0$, and it is equal to vector $\boldsymbol{a}_1$ when $k = 1$. Vector $\boldsymbol{b}$ determines the path from $\boldsymbol{a}_0$ to $\boldsymbol{a}_1$ in the coefficient vector space and the shape transition depends on this vector. Let us call vector $\boldsymbol{b}$ as a deformation path vector. Then, the potential energy $U(\boldsymbol{a})$ is represented as a function of parameter k and vector $\boldsymbol{b}$, and its maximum $U_{\max}$ is represented as a function of vector $\boldsymbol{b}$. Furthermore, geometric constraints which should be satisfied during its manipulation can be also described as a function of vector $\boldsymbol{b}$. The shape transition of a linear object can be then derived by computing coefficient vector $\boldsymbol{b}$ that minimizes the maximum potential energy under geometric constraints. Paths of all points on the object are also generated by solving this minimization problem. It seems that the manipulation without excessive deformation can be realized if some points on the object are moved along their paths which are generated above.

The more exact shape transition can be computed if coefficient vector $\boldsymbol{a}$ is represented by use of vector $\boldsymbol{b} = \left[\ \boldsymbol{b}_1{}^{\mathrm{T}},\ \ \boldsymbol{b}_2{}^{\mathrm{T}},\ \ \cdots,\ \ \boldsymbol{b}_n{}^{\mathrm{T}}\ \right]^{\mathrm{T}}$ as follows:

$$\begin{aligned}\boldsymbol{a} &= (1-k)\boldsymbol{a}_0 + k\boldsymbol{a}_1 \\ &\quad + \sum_{i=1}^{n} k^i (1-k)\boldsymbol{b}_i \quad (0 \le k \le 1).\end{aligned} \tag{11}$$

Numerical Example of Path Generation

In this section, we will show an numerical example in order to demonstrate the effectiveness of our proposed method in the above section. Figure 4 shows an example of required operation. The shape of a linear object in the initial state is shown in Figure 4(a) and that in the objective state is shown in Figure 4(b). In this example, we assume that a linear object has no torsional deformation and that its gravitational energy is negligible. Namely, potential energy consists of flexural energy alone; $U = U_f$. In this case, it is found that angles ϕ and ψ are constantly zero. This implies that the linear object is deformed in $x-z$ plane.

As the angle of left endpoint of the object is fixed and the object must avoid interference with an obstacle which is illustrated in the right side of Figure 4(a) and (b) during its manipulation, geometric constraints imposed on the object can be represented as follows:

$$\int_0^1 \{\theta(0)\}^2\, dk = 0 \tag{12}$$

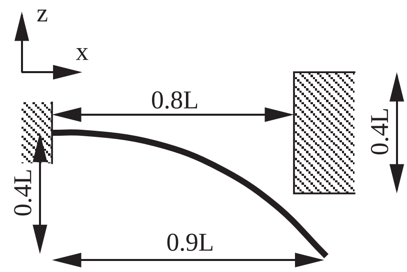
$$\begin{aligned}x(s) \le 0.8L, \quad &\forall \ \{\ s \mid -0.2L \le z(s) \le 0.2L\ \}, \\ &\forall\ k \in [\ 0,\ \ 1\ ].\end{aligned} \tag{13}$$

Let us describe coefficient vector $\boldsymbol{a}$ which represents the shape of the object while its manipulation by use of deformation path vector $\boldsymbol{b} = \left[\ \boldsymbol{b}_1{}^{\mathrm{T}},\ \ \boldsymbol{b}_2{}^{\mathrm{T}},\ \ \boldsymbol{b}_3{}^{\mathrm{T}}\ \right]^{\mathrm{T}}$ as follows:

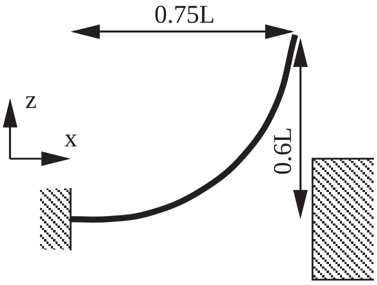
$$\begin{aligned}\boldsymbol{a} &= (1-k)\boldsymbol{a}_0 + k\boldsymbol{a}_1 \\ &\quad + \sum_{i=1}^{3} k^i (1-k)\boldsymbol{b}_i \quad (0 \le k \le 1).\end{aligned} \tag{14}$$

Figure 5 shows the shape transition generated by solving the minimization problem with respect to this vector $\boldsymbol{b}$ and Figure 6 shows the position and the direction of the right endpoint of the object in this shape transition.

We can manipulate this linear object without excessive deformation if its shape transition can be controlled as shown in Figure 5. However, for example, even if we control the position and the direction of only right endpoint as shown in Figure 6, our proposed method does not guarantee that the actual shape transition corresponds to that shown in Figure 5. Therefore, it is very important for realization of optimal manipulation to determine the number of points on a linear object which should be controlled and to determine the position of these points.



(a) Initial state



(b) Objective state

Figure 4: Example of required operation for path generation

4. EXPERIMENTAL RESULT

In this section, we will show an experimental result of the shape measurement in order to demonstrate the validity of our proposed methods.

Let us try to realize the shape transition as shown in Figure 5 by controlling only right endpoint of a linear object. Then, it should be certified that the shape corresponds to that shown in Figure 5 when geometric constraints as shown in Figure 6 are imposed on its right endpoint . The former is computed by use of the method for deformation modeling and the latter is derived by use of the method for path generation. Let us call the result from the path generation method *computation 1* and that from the deformation modeling method *computation 2*. It should be also guaranteed that both of them, especially computation 2, correspond to the actual shape when the same constraints are imposed on the same point.

Let us measure the deformation of a sheet of vinyl chloride 100(mm) long, 12(mm) wide, and 0.5(mm) thick. It is not needed to measure the flexural rigidity of this sheet because its shape is independent of the flexural rigidity in this case.

Three kinds of values, computation 1, computation 2, and the result of measurement, are plotted in Figure 7. The difference between computation 2 and the measurement results from the discrepancy between the given value and the actual value of the

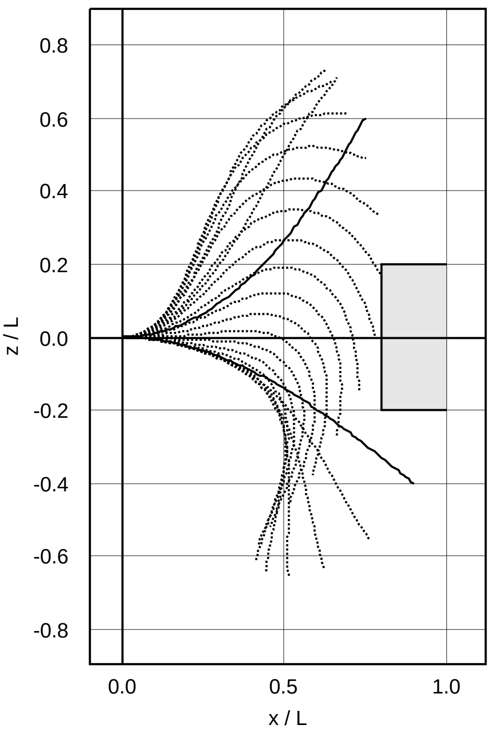


Figure 5: Result of path generation

angle of the left endpoint of the object. It seems that the maximum potential energy is almost minimum in computation 2 because the difference between computation 1 and computation 2 is small. If this difference becomes too large, a few more points on the object must be controlled in order to realize the optimal manipulation. In this case, we can manipulate this object without excessive deformation by controlling the position and the direction of only right endpoint as shown in Figure 6.

5. CONCLUDING REMARKS

A systematic approach to the modeling of linear object deformation and the generation of the optimal path for its manipulation has been developed. First, a method to compute the shape of a linear object was proposed. The shape of the object in the stable state can be computed by minimizing its potential energy under geometric constraints. A procedure for this shape computation was developed by applying nonlinear programming technique. Some numerical examples proved that this procedure had a capability of computing the large bending and torsional deformation. Secondly, a method to generate the optimal path for linear object manipulation was proposed. It was assumed that its maximum potential energy becomes minimum on the most suitable path. This optimal path can be computed by applying our deformation modeling method. Finally, the validity of these methods was demonstrated with a measuring experiment. It was shown that the determination of the number of points to control and their position on the object was important to realize optimal manipulation.

We can evaluate the shape of a deformed linear object in three-dimensional space under various conditions on a computer using our approach.

In this paper, the optimal manipulation path is generated by minimizing the maximum potential energy, but it can be also determined considering another value, for example, the stress. By applying our proposed method, The optimal path can be also derived even in such case.

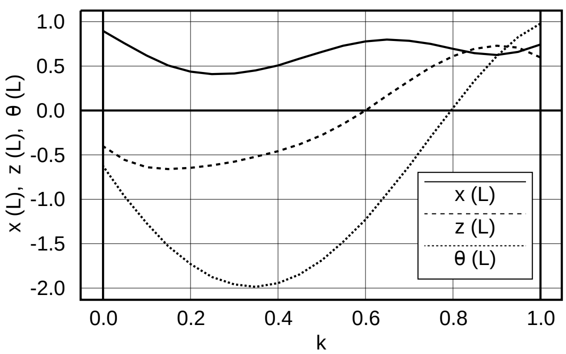


Figure 6: Positional and directional control values for optimal path

6. ACKNOWLEDGEMENT

This study has been supported by a Grant-in-Aid for Scientific Research of Japan Society for the Promotion of Science No.12750214 in 2000.

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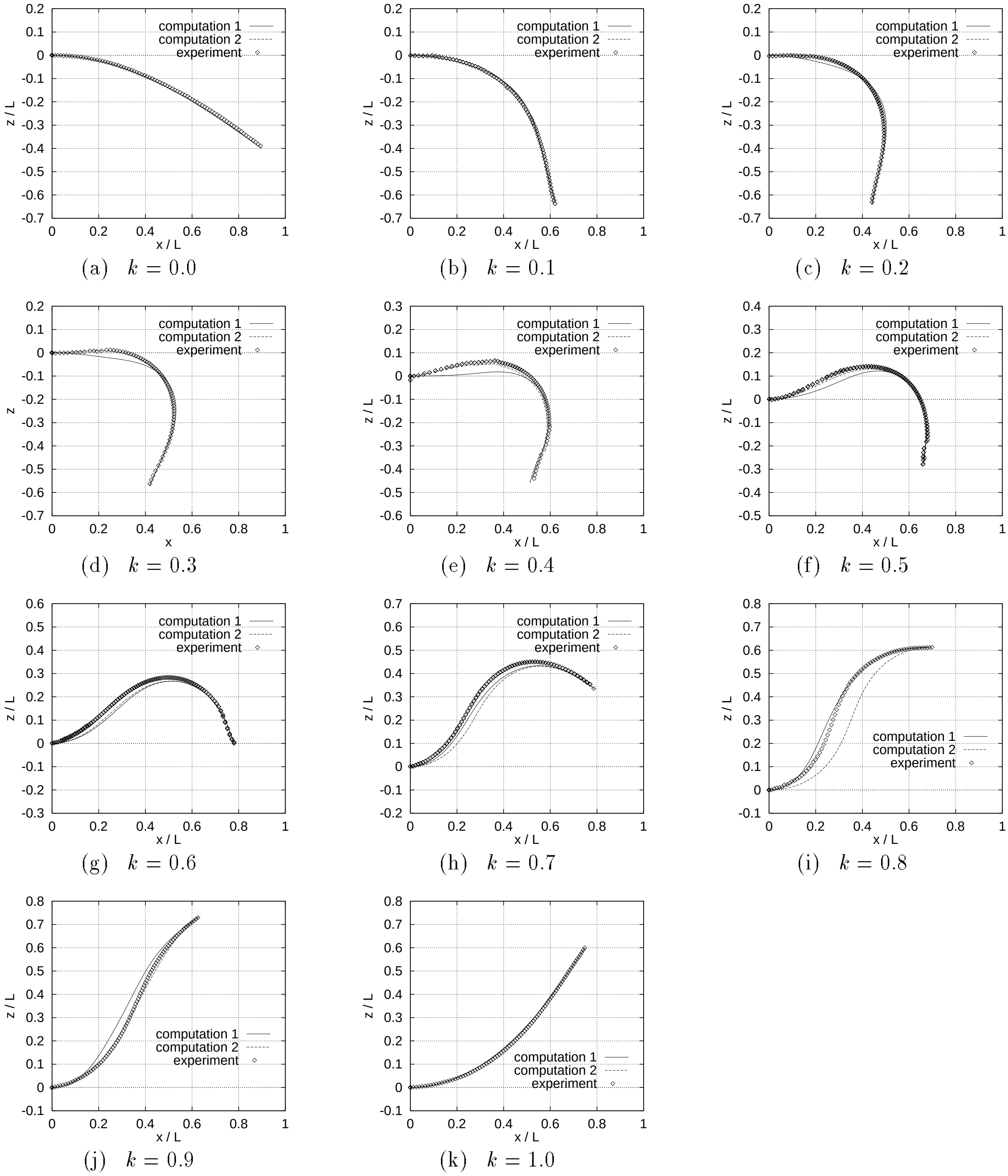

Figure 7: Comparison among computed shapes using path generation method, those using deformation modeling method, and experimental results