Indirect Simultaneous Positioning of Deformable Objects by Redundant Fingers without Physical Parameters

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ABSTRACT

This paper describes the control of indirect simultaneous positioning of a viscoelastic 2D object without any physical parameters. Applying continua modeling of isotropic viscoelastic deformation, I first show that the positioning can be performed successfully by a simple I control without physical parameters. Then, I show that a redundant system, where the number of manipulated displacements exceeds the number of positioned displacements, performs the positioning successfully.

KEY WORDS
Deformation, Positioning, Control, Redundancy

1 Introduction

Many manipulative operations that deal with deformable objects result in a positioning of multiple points on a deformable object [1, 2]. In this positioning, multiple points on a deformable object should be guided to their desired locations simultaneously. Moreover, it is often impossible to manipulate the positioned points directly. For example, one operation called linking is popular in garment manufacturing. In linking of fabrics, loops at the end of a fabric must be matched to loops of another fabric so that the two fabrics can be sewed seamlessly. These points cannot be manipulated directly since a sewing needle is guided along the matched loops. Mating of a flexible part in electric industry also results in the positioning of mated points on the object. These points cannot be manipulated directly since the points in a mating part contact with a mated part. Consequently, we find that a positioning of multiple points on a deformable object is one of fundamental operations in the manipulation of deformable objects. Since the positioned points cannot be manipulated directly, the guidance of positioned points must be performed by controlling some points except the positioned points, as illustrated in Figure 1. This operation is referred to as *indirect simultaneous positioning*, which is abbreviated as ISP.

An iterative control law based on a roughly estimated physical model of an extensible object has been proposed [3]. It has experimentally shown that the positioning can be performed successfully despite of the discrepancy of physical parameters between an actual object and its model. Simple PID-control has been successfully applied to the ISP [4]. Unfortunately, theoretical analysis has not studied yet as a particle-based model has been applied to the derivation of the control law and the modeling of a deformable object. In this paper, I will apply continua modeling of a viscoelastic object to the indirect simultaneous positioning and will show that a simple I-control performs the positioning successfully.

2 Description of Indirect Simultaneous Positioning

Let us divide a deformable object into a set of triangles or tetrahedra. Then, the object deformation can be described by a set of nodal points. Assume that positioned points and manipulated points are involved in the nodal points. Let $\boldsymbol{u}_i = [u_i, v_i]^T$ be the displacement vector of nodal

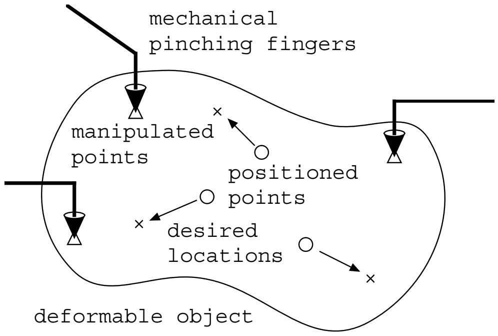
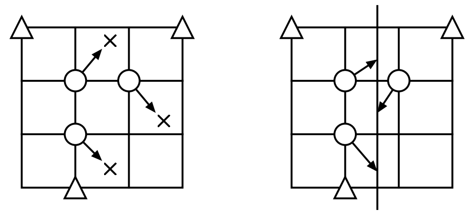


Figure 1: Indirect simultaneous positioning of deformable object

point P_i. Some displacements of nodal points should be guided to their desired values in an ISP. These displacements are referred to as *positioned displacements*. This guidance should be performed by controlling some displacements except positioned displacements. These displacements are referred to as *manipulated displacements*. Displacements except positioned displacements or manipulated displacements are referred to as *non-positioned non-manipulated displacements*. Consequently, we can classify a set of displacements into three subsets; 1) manipulated displacements, 2) positioned displacements, and 3) non-positioned non-manipulated displacements. For example, three points marked as circles should be guided to their desired locations marked as crosses in a positioning illustrated in Figure 2-(a). This guidance is performed by controlling three points marks as triangles. Thus, a set of positioned displacements is given by u_5, v_5, u_6, v_6, u_{10}, and v_{10} while a set of manipulated displacements is given by u_3, v_3, u_{12}, v_{12}, u_{15}, and v_{15}. The desired values of positioned displacements can be computed from the initial coordinates and the desired coordinates of positioned points. In a positioning illustrated in Figure 2-(b), three points marked as circles should be aligned on a target line perpendicular to the x-axis. Note that we must guide the x-coordinate of the three points to the x-intercept of the line, while we do not have to control the y-coordinate of the three points. Thus, a set of positioned displacements in this example is given by u_5, u_6, and u_{10}. Displacements v_5, v_6, and v_{10} are involved in non-positioned non-manipulated displacements. The desired values of positioned displacements can be computed from the initial x-coordinate of positioned points and the x-intercept of the target line.



(a) guidance to desired points (b) guidance to desired line

Figure 2: Manipulated, positioned, and non-positioned non-manipulated displacements

Let $\boldsymbol{r}_m$ be a vector consisting of manipulated displacements, $\boldsymbol{r}_p$ be a vector composed of positioned displacements, and $\boldsymbol{r}_n$ be a vector consisting of non-positioned non-manipulated displacements. Vectors $\boldsymbol{r}_m$, $\boldsymbol{r}_p$, and $\boldsymbol{r}_n$ are referred to as *manipulated displacement vector*, *positioned displacement vector*, and *non-positioned non-manipulated displacement vector*, respectively. Let m, p, and n be dimension of vector $\boldsymbol{r}_m$, that of vector $\boldsymbol{r}_p$, and that of vector $\boldsymbol{r}_n$, respectively. For example, in a positioning shown in Figure 2-(a), we have

$$\begin{aligned}\boldsymbol{r}_m &= [u_5, v_5, u_6, v_6, u_{10}, v_{10}]^T,\\ \boldsymbol{r}_p &= [u_3, v_3, u_{12}, v_{12}, u_{15}, v_{15}]^T,\\ \boldsymbol{r}_n &= [u_0, v_0, u_1.v_1, \cdots, u_{14}, v_{14}]^T.\end{aligned}$$

Dimensions are given by $m = 6$, $p = 6$, and $n = 24$. In a positioning shown in Figure 2-(b), we have

$$\begin{aligned}\boldsymbol{r}_m &= [u_5, u_6, u_{10}]^T,\\ \boldsymbol{r}_p &= [u_3, v_3, u_{12}, v_{12}, u_{15}, v_{15}]^T,\\ \boldsymbol{r}_n &= [v_5, v_6, v_{10}, u_0, v_0, \cdots, u_{14}, v_{14}]^T.\end{aligned}$$

Dimensions are given by $m = 3$, $p = 6$, and $n = 27$.

Recall that individual positioned displacements should be guided to their desired values. This implies that all elements composing vector $\boldsymbol{r}_p$ have their desired values. Let $\boldsymbol{r}_p^*$ be a vector consisting of the desired values of the positioned displacements. Then, the goal of indirect simultaneous positioning is given by an equation; $\boldsymbol{r}_p = \boldsymbol{r}_p^*$.

This goal must be achieved by controlling manipulated displacements, $\boldsymbol{r}_m$.

3 Control Law

Assume that a vision system can measure the current values of positioned displacements. This implies that the current value of positioned displacement vector $\boldsymbol{r}_p$ can be measured through a vision system. Moreover, recall that mechanical fingers pinch an extensible object and no slip between the fingers and the object occurs. Namely, the current value of manipulated displacement vector $\boldsymbol{r}_m$ can be computed from the motion of mechanical pinching fingers.

Let us define a mapping from a set of positioned displacements to a set of manipulated displacements. Let r_i be a positioned displacement and r_i^* be its goal displacement. Determine a manipulated displacement r_j corresponding to each positioned displacement r_i. Then, let us apply the following simple control law:

$$r_j = K_I \int_0^t (r_i^* - r_i)\ \mathrm{d}t, \tag{1}$$

where K_I denotes integral gain. This equation computes the commanded values of manipulated displacements r_j.

The above equation provides a continuous control law. Let us derive a discrete control law. Assume that positioned displacement r_i can be measured at time interval T. Let r_i^k and r_j^k be the positioned displacement and the manipulated displacement at the k-th time interval $[kT, (k+1)T]$. Then, the above continuous control law turns into a discrete control law as follows:

$$r_j^{k+1} = r_j^k + K_I(r_i^* - r_i^k). \tag{2}$$

Namely, the commanded value of manipulated displacement r_j^{k+1} at the next time interval is computed from the current value of manipulated displacement r_j^k and the current error of positioned displacement $r_i^* - r_i^k$. Note that the these control laws include no physical parameters of a positioned object. This implies that no identification of physical parameters is needed.

4 Simulation

4.1 Dynamic modeling of 2D viscoelastic deformation

Let $\boldsymbol{\sigma}$ be a pseudo stress vector and $\boldsymbol{\varepsilon}$ be a pseudo strain vector. Stress-strain relationship of 2D isotropic viscoelastic deformation is formulated as follows:

$$\boldsymbol{\sigma} = (\lambda I_\lambda + \mu I_\mu)\boldsymbol{\varepsilon} \tag{3}$$

where

$$\lambda = \lambda^{\mathrm{ela}} + \lambda^{\mathrm{vis}}\frac{\mathrm{d}}{\mathrm{d}t}, \qquad \mu = \mu^{\mathrm{ela}} + \mu^{\mathrm{vis}}\frac{\mathrm{d}}{\mathrm{d}t}.$$

Elasticity of the object is specified by two elastic moduli λ^{ela} and μ^{ela} while its viscosity is specified by two viscous moduli λ^{vis} and μ^{vis}. Matrices I_λ and I_μ are matrix representations of isotropic tensors, which are given as follows in 2D deformation:

$$I_\lambda = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad I_\mu = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The stress-strain relationship can be converted into a relationship between a set of forces applied to nodal points and a set of displacements of the points. Let $\boldsymbol{u}_N$ be a set of displacements of nodal points. Let J_λ and J_μ are connection matrices, which can be geometrically determined by object coordinate components of nodal points. Replacing I_λ by J_λ, I_μ by J_μ, and $\boldsymbol{\varepsilon}$ by $\boldsymbol{u}_N$ in the stress-strain relationship (3) of a viscoelastic object yields a set of viscoelastic forces applied to nodal points as follows:

$$\text{viscoelastic force} \quad = \quad (\lambda J_\lambda + \mu J_\mu)\boldsymbol{u}_N. \tag{4}$$

Introducing $\boldsymbol{v}_N = \dot{\boldsymbol{u}}_N$, we have

$$\text{viscoelastic force} = K\boldsymbol{u}_N + B\boldsymbol{v}_N \tag{5}$$

where

$$K = \lambda^{\mathrm{ela}} J_\lambda + \mu^{\mathrm{ela}} J_\mu, \quad B = \lambda^{\mathrm{vis}} J_\lambda + \mu^{\mathrm{vis}} J_\mu.$$

Let M be an inertia matrix and $\boldsymbol{f}$ be a set of external forces applied to nodal points. Let us describe a set of geometric constraints imposed on the nodal points by

$A^T \boldsymbol{u}_N = \boldsymbol{b}$. The number of columns of matrix A is equal to the number of geometric constraints. Let $\boldsymbol{\lambda}$ be a set of constraint forces corresponding to the geometric constraints. A set of dynamic equations of nodal points is then given by

$$M\dot{\boldsymbol{v}}_N = -K\boldsymbol{u}_N - B\boldsymbol{v}_N + \boldsymbol{f} + A\boldsymbol{\lambda}.$$

Applying the constraint stabilization method [5] to the constraints specified by angular velocity ω, system dynamic equations are described as follows:

$$\begin{aligned} \dot{\boldsymbol{u}}_N &= \boldsymbol{v}_N, \\ M\dot{\boldsymbol{v}}_N - A\boldsymbol{\lambda} &= -K\boldsymbol{u}_N - B\boldsymbol{v}_N + \boldsymbol{f}, \\ -A^T\dot{\boldsymbol{v}}_N &= 2\omega A^T\boldsymbol{v}_N + \omega^2 A^T(\boldsymbol{u}_N - \boldsymbol{b}). \end{aligned}$$

Consequently,

$$\begin{bmatrix} I & & \\ & M & -A \\ & -A^T & \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{u}}_N \\ \dot{\boldsymbol{v}}_N \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_N \\ -K\boldsymbol{u}_N - B\boldsymbol{v}_N + \boldsymbol{f} \\ 2\omega A^T\boldsymbol{v}_N + \omega^2 A^T(\boldsymbol{u}_N - \boldsymbol{b}) \end{bmatrix}. \tag{6}$$

Note that the above linear equation is solvable since the matrix is regular, implying that we can sketch $\boldsymbol{u}_N$ and $\boldsymbol{v}_N$ using numerical solver such as the Euler method or the Runge-Kutta method.

4.2 Dynamic simulation of indirect simultaneous positioning

Let us simulate an indirect simultaneous positioning by taking a simple example illustrated in Figure 3. Two-dimensional deformation of a viscoelastic object is described by nodal points P_0 through P_{15}. Let us guide three points P_5, P_6, and P_{10} to their desired location by controlling three points P_3, P_4, and P_{14}. Positioned displacement vector is given by $\boldsymbol{r}^p = [u_5, v_5, u_6, v_6, u_{10}, v_{10}]^T$ and manipulated displacement vector is $\boldsymbol{r}^m = [u_3, v_3, u_4, v_4, u_{14}, v_{14}]^T$. Let us introduce a distance-based mapping from the positioned displacements to the manipulated displacements. Control law is then formulated as follows:

$$\boldsymbol{u}_3 = K_I \int_0^t (\boldsymbol{u}_6^* - \boldsymbol{u}_6)\ \mathrm{d}t,$$

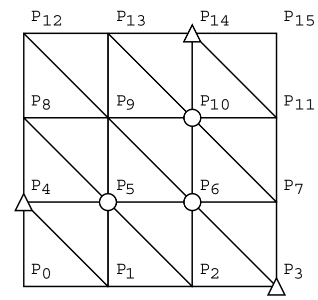


Figure 3: Viscoelastic 2D object for simulation

$$\boldsymbol{u}_4 = K_I \int_0^t (\boldsymbol{u}_5^* - \boldsymbol{u}_5)\ \mathrm{d}t,$$
$$\boldsymbol{u}_{14} = K_I \int_0^t (\boldsymbol{u}_{10}^* - \boldsymbol{u}_{10})\ \mathrm{d}t.$$

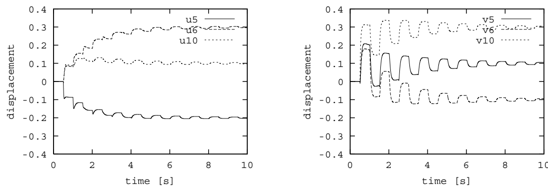
The corresponding discrete control law is given by

$$\begin{bmatrix} u_3^{k+1} \\ v_3^{k+1} \end{bmatrix} = \begin{bmatrix} u_3^k \\ v_3^k \end{bmatrix} + K_I \begin{bmatrix} u_6^* - u_6^k \\ v_6^* - v_6^k \end{bmatrix},$$
$$\begin{bmatrix} u_4^{k+1} \\ v_4^{k+1} \end{bmatrix} = \begin{bmatrix} u_4^k \\ v_4^k \end{bmatrix} + K_I \begin{bmatrix} u_5^* - u_5^k \\ v_5^* - v_5^k \end{bmatrix},$$
$$\begin{bmatrix} u_{14}^{k+1} \\ v_{14}^{k+1} \end{bmatrix} = \begin{bmatrix} u_{14}^k \\ v_{14}^k \end{bmatrix} + K_I \begin{bmatrix} u_{10}^* - u_{10}^k \\ v_{10}^* - v_{10}^k \end{bmatrix}.$$

Elastic and viscous moduli are $\lambda^{\mathrm{ela}} = 7.0$, $\lambda^{\mathrm{vis}} = 4.0$, $\mu^{\mathrm{ela}} = 5.0$, and $\mu^{\mathrm{vis}} = 2.0$. Density is given by $\rho = 0.2$. Positioned displacements are measured at time interval $T = 0.5$. Let desired values of the positioned displacements be

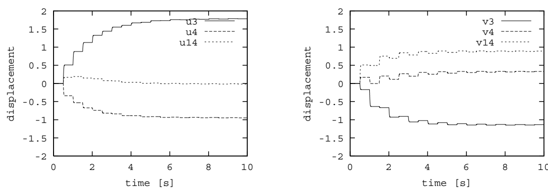
$$\boldsymbol{u}_5^* = \begin{bmatrix} -0.20 \\ 0.10 \end{bmatrix},\ \boldsymbol{u}_6^* = \begin{bmatrix} 0.30 \\ -0.10 \end{bmatrix},\ \boldsymbol{u}_{10}^* = \begin{bmatrix} 0.10 \\ 0.30 \end{bmatrix}.$$

Motion of the positioned displacements is plotted in Figure 4. Gain is given by $K_I = 1.7$. Vibration comes from the viscoelastic nature of the object. Despite of the vibration, the positioned displacements converge to their desired values, as shown in the figure. Motion of the manipulated displacements is plotted in Figure 5. As shown in the figure, the manipulated displacements are updated at every time interval. Deformed shapes during the positioning process are described in Figure 6. Crosses in the



(a) u (b) v

Figure 4: Motion of positioned points



(a) u (b) v

Figure 5: Motion of manipulated points

figures denote the desired values of the positioned displacements. As shown in the figure, the positioned displacements converge to their desired values.

Let us guide the x-coordinates of P_5, P_6, and P_{10} to their desired values by controlling the x- and y-coordinates of P_3, P_4, and P_{14}. Namely, a set of manipulated displacements is redundant. Positioned displacement vector is given by $\boldsymbol{r}^p = [u_5, u_6, u_{10}]^T$ and manipulated displacement vector is $\boldsymbol{r}^m = [u_3, v_3, u_4, v_4, u_{14}, v_{14}]^T$. Let us introduce the discrete control law given by

$$\begin{bmatrix} u_3^{k+1} \\ v_3^{k+1} \end{bmatrix} = \begin{bmatrix} u_3^k \\ v_3^k \end{bmatrix} + K_I \begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix} (u_6^* - u_6^k),$$
$$\begin{bmatrix} u_4^{k+1} \\ v_4^{k+1} \end{bmatrix} = \begin{bmatrix} u_4^k \\ v_4^k \end{bmatrix} + K_I \begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix} (u_5^* - u_5^k),$$
$$\begin{bmatrix} u_{14}^{k+1} \\ v_{14}^{k+1} \end{bmatrix} = \begin{bmatrix} u_{14}^k \\ v_{14}^k \end{bmatrix} + K_I \begin{bmatrix} \alpha \\ 1-\alpha \end{bmatrix} (u_{10}^* - u_{10}^k),$$

Deformed shapes during the positioning process at $\alpha = 0.7$ are described in Figure 7. As shown in the figure, the positioned displacements converge to their desired values. It turns out that the positioned displacements do not converge to their desired values unless $\alpha \geq 0.5$.

5 Concluding Remarks

I have applied continua modeling of a viscoelastic object to the indirect simultaneous positioning and have simulated the positioning process. I have shown that a simple I-control performs the positioning successfully without any physical parameters. In addition, I have shown that a redundant system, where the number of manipulated displacements exceeds the number of positioned displacements, performs the positioning successfully. I am going to investigate the stability of the positioning process based on the continua modeling of viscoelastic deformation. Experimental verification is also a future issue.

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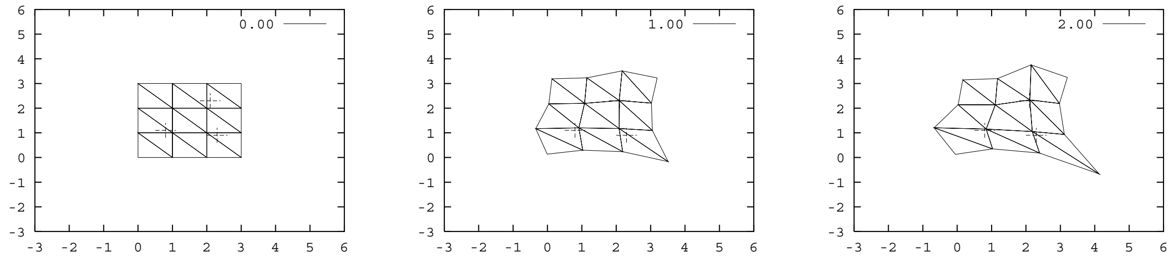

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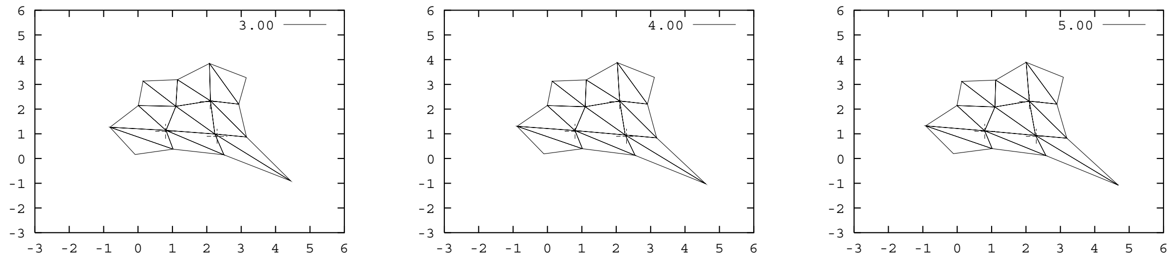

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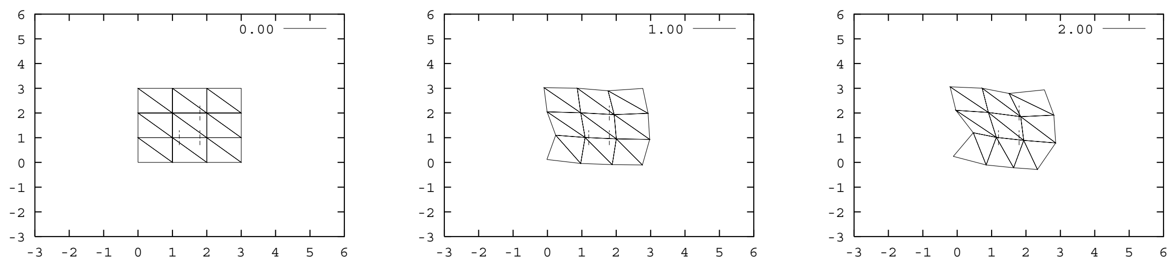
Figure 6: Process of indirect simultaneous positioning to desired points



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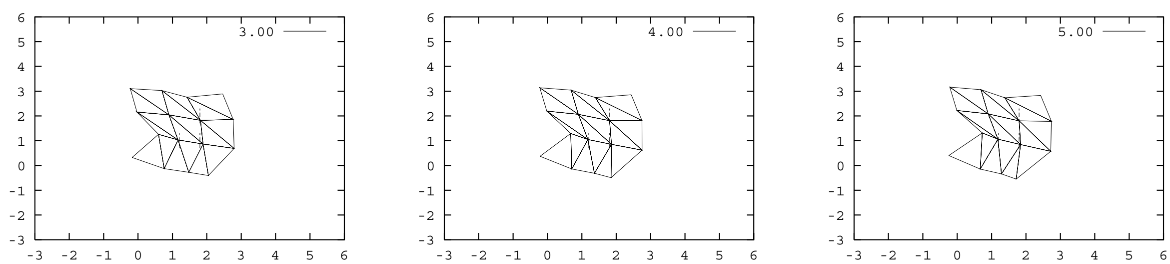

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Figure 7: Process of indirect simultaneous positioning by redundant system