

# Measurement and Modeling of Dynamic Viscoelasticity and Surface Stickiness

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**Abstract:** This manuscript focuses on dynamic deformation of viscoelastic materials. Analysis of dynamic behavior of soft robots requires dynamic modeling of soft robot materials. We formulate dynamic deformation of viscoelastic materials via power laws. Model parameters in power laws are identified by numerically minimizing the error between measured and calculated stress–strain relationships. Additionally, we experimentally evaluate surface stickiness of viscoelastic materials.

**Keywords:** soft robot, dynamic deformation, viscoelasticity, modeling, surface stickiness

## 1. INTRODUCTION

Various viscoelastic materials are widely applied to soft robots. Soft robots deform during their motion. Thus, we have to obtain dynamic deformation properties of soft robot materials. In order to analyze the behavior of soft robots, viscoelastic materials often exhibit nonlinearity and hysteresis. Currently, a unified method for modeling such dynamic deformation properties of viscoelastic materials has not been established. In this manuscript, we propose a method to model the dynamic deformation properties of viscoelastic materials. Additionally, we focus on surface stickiness. Viscoelastic materials often exhibit stickiness on their surfaces. We thus experimentally evaluate surface stickiness of viscoelastic materials.

## 2. MEASUREMENT OF STRESS–STRAIN RELATIONSHIPS

In order to obtain the dynamic stress–strain relationship of the material, we apply predetermined displacement to a material sample and measure the load exerted to the sample during the motion. Displacement is provided by a probe pushing the sample. Note that the contact between the probe and the sample is maintained during the measurement. Assume that the sample be a cylindrical shape. Strain is then obtained by dividing the displacement value by the sample thickness, and stress is obtained by dividing the load value by the contact area between the sample and the probe. Figure 1 shows the measurement process. We used a force–displacement measurement unit FSA-1KE-50N (Imada Co., Ltd.). We determined the vertical motion of the probe so that the sample and the probe were always in contact.

The stress–strain relationship was measured for Haptics of Wonder [1] (Taica Co., Ltd.). Haptics of Wonder consists of twelve samples made from  $\alpha$  gel. Each sample has different deformation and surface properties. We obtain stress and strain during a given time period, followed by the stress–strain relationship of the samples.

<sup>†</sup> Shinichi Hirai is the presenter of this paper.



Fig. 1 Pushing test of soft material samples

## 3. MODELING OF VISCOELASTIC PROPERTIES

In general, the relationship between stress and strain in soft materials is nonlinear. Regarding the elastic properties of elastomers, nonlinear models such as the Mooney–Rivlin model and the Ogden model have been proposed [2]. On the other hand, a dynamic model is necessary for the motion of soft robots and manipulation of soft objects rather than static models given in nonlinear elastic models. Therefore, we should formulate not only elastic properties but also viscous properties.

In this manuscript, we assume that the elasticity and viscosity follow the power law. That is, the magnitude of the stress due to the elastic element is proportional to the magnitude of the strain  $\varepsilon$  to the power of  $E_p$ . Similarly, the magnitude of the stress due to the viscous element is proportional to the strain rate  $\dot{\varepsilon}$  to the power of  $C_p$ . Then, the magnitude of the stress due to the elastic element is described as  $|\varepsilon|^{E_p}$ , and the sign of the stress is described as  $\text{sgn}(\varepsilon)$  using the sign function. Letting  $E$  be the proportional constant, the stress due to the elasticity is formulated as  $E \text{sgn}(\varepsilon)|\varepsilon|^{E_p}$ . Similarly, letting  $c$  be the proportional constant of viscosity, the stress due to the viscous element is formulated as  $c \text{sgn}(\dot{\varepsilon})|\dot{\varepsilon}|^{C_p}$ . Con-

sequently, the stress caused by a viscoelastic element at time  $t$  is the sum of the stress due to the elastic element and the stress due to the viscous element:

$$\sigma(t) = E \operatorname{sgn}(\varepsilon)|\varepsilon|^{E_p} + c \operatorname{sgn}(\dot{\varepsilon})|\dot{\varepsilon}|^{c_p} \quad (1)$$

Note that the above equation is characterized by four deformation parameters:  $E$ ,  $E_p$ ,  $c$ , and  $c_p$ .

To simulate the measurement of the stress–strain relationship described in the previous section, we assume that the strain is given by a sine wave with amplitude  $A$ , angular frequency  $\omega$ , phase  $\phi$ , and bias  $b$ :

$$\varepsilon = A \sin(\omega t + \phi) + b \quad (2)$$

Since inertial force acts on the sample, the dynamic stress–strain relationship is described as follows:

$$E \operatorname{sgn}(\varepsilon)|\varepsilon|^{E_p} + c \operatorname{sgn}(\dot{\varepsilon})|\dot{\varepsilon}|^{c_p} + \rho \ddot{\varepsilon} = \sigma(t) \quad (3)$$

where  $\rho$  denotes the linear density of the material sample. By substituting Eq. (2) into Eq. (3), we can calculate the stress applied to a material sample, yielding the stress–strain relationship of the material.

#### 4. IDENTIFICATION OF MODEL PARAMETERS

We identify the values of model parameters  $E$ ,  $E_p$ ,  $c$ ,  $c_p$ , and  $\rho$  from measurement results. Let  $\varepsilon_m(t)$  and  $\sigma_m(t)$  be the strain and stress calculated from the measured displacements and loads at time  $t$ . Let  $[0, T]$  be the range of the measurement time  $t$ . First, we obtain motion parameters  $A$ ,  $\omega$ ,  $\phi$ , and  $b$  in Eq. (2). Comparing the measured values  $\varepsilon_m(t)$  and Eq. (2), values of the motion parameters are calculated through curve fitting. Using the obtained motion parameter values and Eq. (2), we calculate  $\varepsilon(t)$ ,  $\dot{\varepsilon}(t)$ , and  $\ddot{\varepsilon}(t)$ .

Next, we identify the values of model parameters  $E$ ,  $E_p$ ,  $c$ ,  $c_p$ , and  $\rho$ . Substituting model parameter values and calculated  $\varepsilon(t)$ ,  $\dot{\varepsilon}(t)$ , and  $\ddot{\varepsilon}(t)$  into Eq. (3), we obtain stress  $\sigma(t)$  at time  $t$ . The error between the calculated value  $\sigma(t)$  and the measured value  $\sigma_m(t)$  is then formulated as follows:

$$D(E, E_p, c, c_p, \rho) = \int_0^T \{\sigma(t) - \sigma_m(t)\}^2 dt \quad (4)$$

Note that the above error is non-negative and depends on model parameters  $E$ ,  $E_p$ ,  $c$ ,  $c_p$ , and  $\rho$ . The smaller the value of the non-negative function  $D$ , the closer the calculated stress is to the measured stress. Consequently, the values of model parameter can be identified by minimizing the above error with respect to the model parameters:

$$\text{minimize } D(E, E_p, c, c_p, \rho) \quad (5)$$

This minimization can be achieved through numerical optimization.

The identified values of model parameters are shown in Table 1. The unit of parameter  $E$  is MPa, the unit of  $c$  is MPa · s, the unit of  $\rho$  is  $10^{-8}$  mbox{g}/mm. Figure 2 shows the stress–strain relationships obtained from the identified values for the samples. The red lines are the measured values, and the green lines originate from the

Table 1 Identified model parameters of samples

	$E$	$E_p$	$c$	$c_p$	$\rho$
#01	3.14	1.67	0.0310	2.28	2.36
#02	1.89	2.01	0.0469	3.19	2.36
#03	1.13	2.06	0.0293	1.98	2.36
#04	1.19	2.11	0.0296	1.26	2.27
#05	0.657	2.12	0.00905	1.09	2.59
#06	0.388	2.31	0.00577	1.03	2.35
#07	0.264	2.40	0.00232	1.02	2.46
#08	0.495	2.51	0.0127	1.51	2.41
#09	0.150	2.37	0.00147	0.519	2.22
#10	0.128	2.29	0.00123	0.708	2.35
#11	0.0749	2.00	0.000653	1.37	2.60
#12	0.542	3.77	0.00274	0.422	2.29

identification result. We find that the model parameters can be identified and that the identified model parameters well represent the measured stress–strain relationship.

The elastic moduli of samples #01 to #05 are evaluated with Asker C and decrease in order of the sample number. The values of the parameter  $E$  of samples #01 to #05 also decrease in order of the sample number, suggesting that the values of the parameter  $E$  correspond to the elastic moduli. The materials of #03 and #04 are the same, resulting that their values of the parameter  $E$  are close to each other. Also, the materials of samples #09 and #10 are the same, but the values of  $E$  of #09 and #10 differ by about 15%. This difference might originate from the change in the contact area between the probe and the sample due to the surface properties.

#### 5. SURFACE STICKINESS

Soft materials often have sticky surfaces. Stickiness measurement was proposed in [3]. Here we apply a simple probe test. We provided the following motion to the probe of the force–displacement measurement unit. Initially, the probe is out of contact with the sample. Then, the probe pushes the sample for a given distance. Finally, the probe moves back to the initial location. Namely, transition from non–contact state to contact state occurs, followed by a transition from contact state to non–contact state. We obtained the stress–strain relationships corresponding to this probe motion.

Figure 3 shows the stress–strain relationships in stickiness evaluation. Negative values in stress originate from the surface stickiness. From the measurements, we find that samples #01 and #02 have small stickiness on their surfaces while samples #08 and #09 have large stickiness on their surfaces. Samples #03 and #04 compose of the same material but have different surface finishing, resulting that sample #03 have more sticky surface than sample #04. Negative stress in sample #03 is somewhat larger than that in sample #04. Similarly, samples #09 and #10 compose of the same material but have different surface finishing, resulting that sample #09 have more sticky surface than sample #10. Obviously, negative stress in sample #09 is larger than that in sample #10. We find that

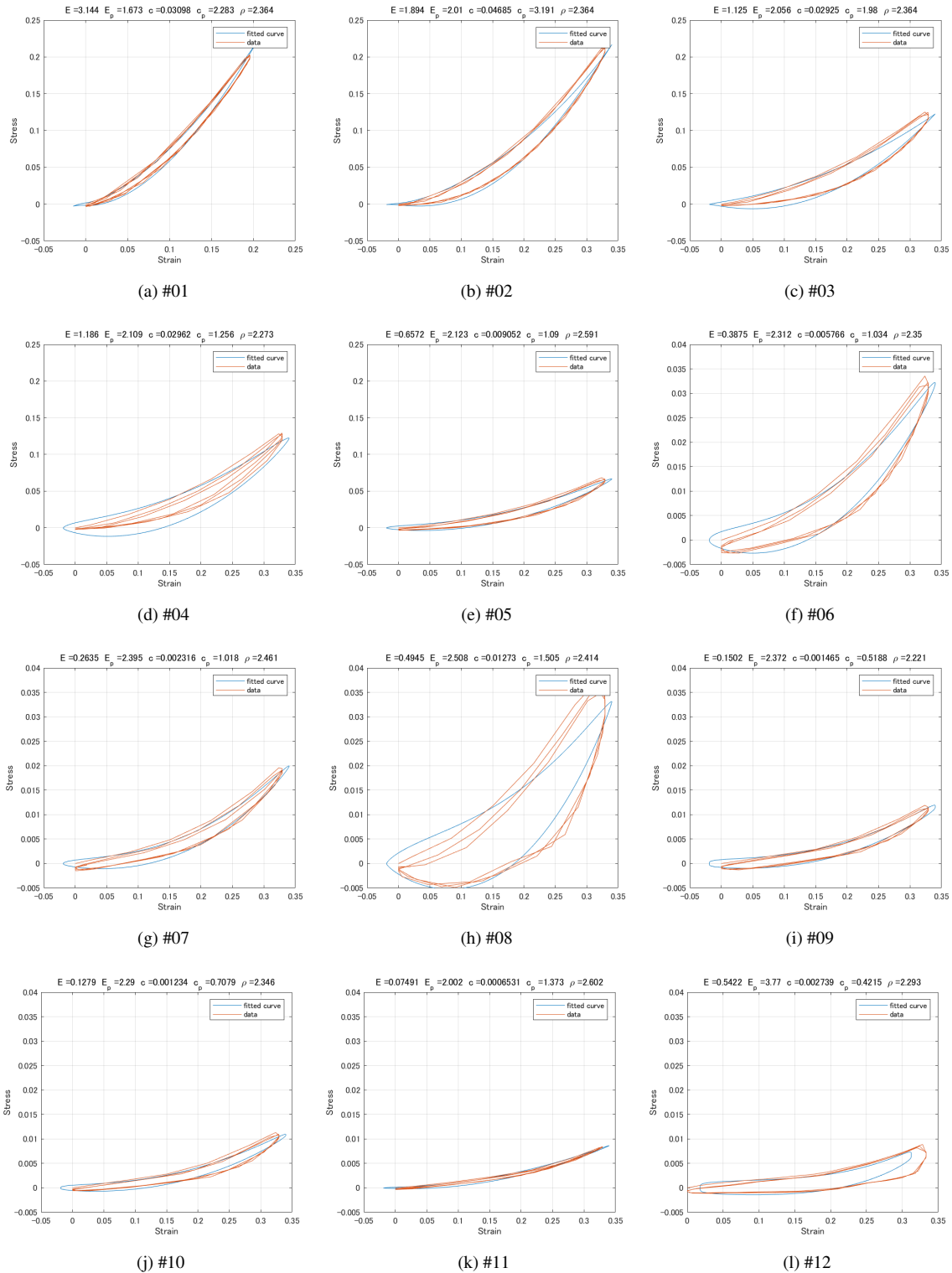


Fig. 2 Estimated stress–strain relationship using identified model parameters

negative stress represents the surface stickiness of viscoelastic materials.

## 6. CONCLUSION

Dynamic deformation properties of viscoelastic materials were represented by power law. Model parameters in stress–strain relationships were identified by numerically minimizing the error function between a measurement

result and a model. Surface stickiness was also evaluated experimentally. Stickiness modeling is a future issue.

Measurement and modeling of dynamic viscoelasticity is important not only in soft robotics but also soft object manipulation. For example, it is required to measure and identify deformation properties in food manipulation. Deformation properties of food items affect their manipulation, suggesting that measuring and modeling

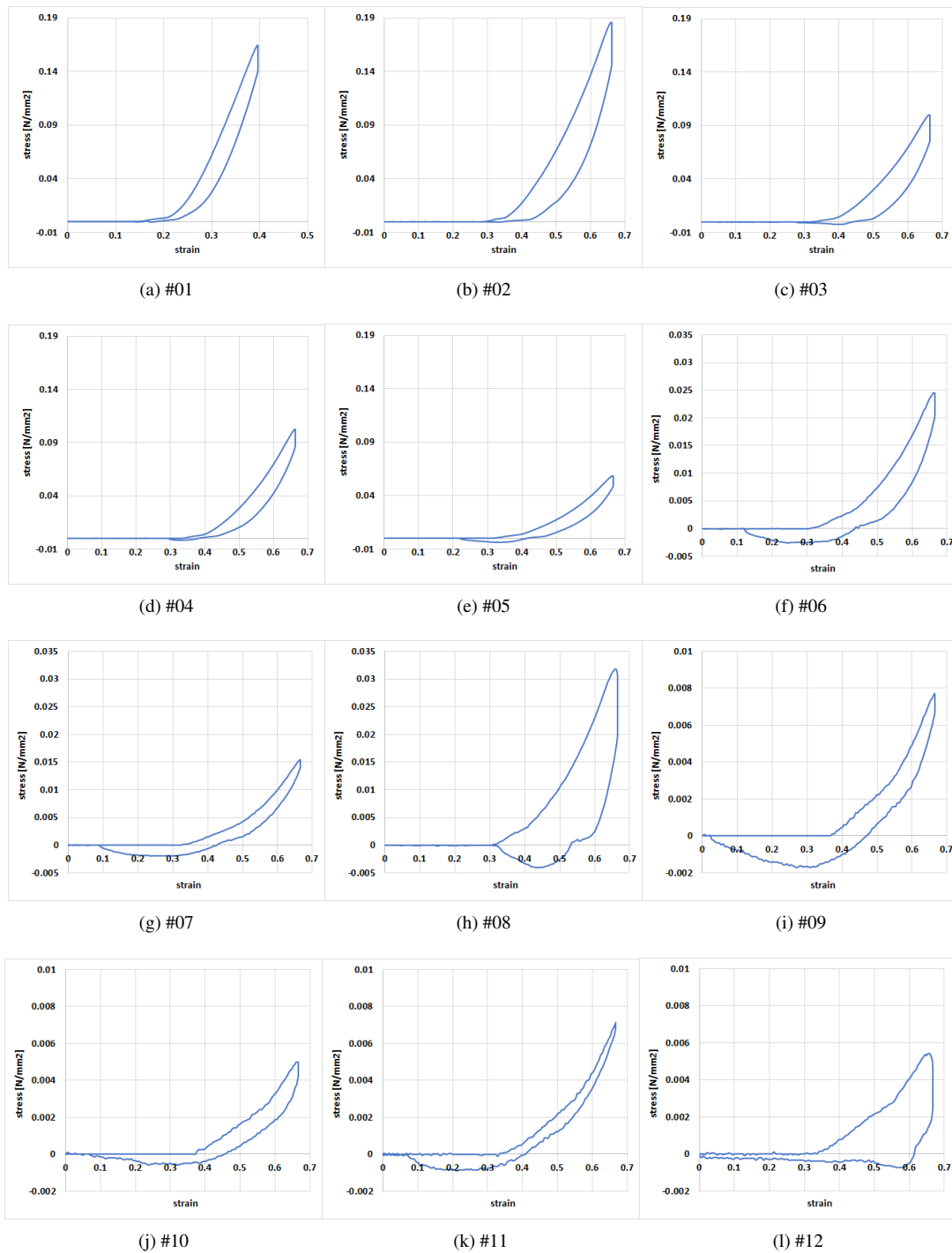


Fig. 3 Measured stress–strain relationship in stickiness evaluation

of their deformation is essential in evaluation of manipulative processes. Surface properties including stickiness and friction also affect object manipulation, implying that measuring and modeling of their surface properties is important for evaluating the manipulative processes.

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