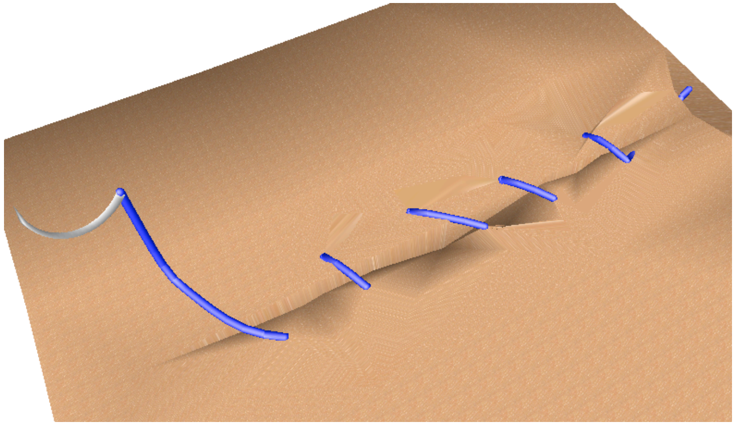
Physics-based DLO Simulation and It's Application in Suturing



Hans Fuhan Shi and Shahram Payandeh

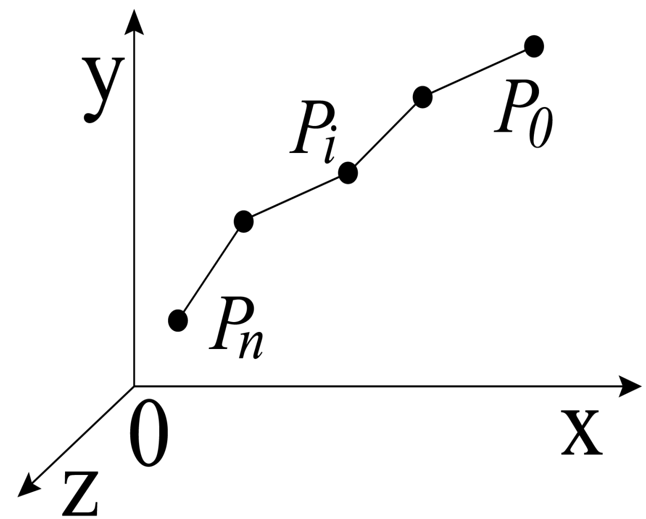
Simon Fraser University
Burnaby, Canada

Outline

- **DLO Model description**
  - External forces
  - Internal forces
  - Force propagation
- **Real-time knotting and unknotting**
  - Collision detection
  - Collision management
- **Knotting and unknotting case study and result**
  - Force study
  - Knotting
  - Unknotting

Model Description

- **Mass Points - physics**
  - A sequence of mass points laying on the centre line of the suture
- **Cylinder - geometry**
  - During graphic rendering, we use cylinder as segment to connect two successive points
- **Forces - Haptic**
  - To computer the configuration of the suture, we need to obtain the net force acting on each point, and then use Euler method to get the positions.



Rope model

Model Description - External Forces

- **Input forces**
  - by the user with the grippers
- **Friction forces**
  - during knotting or unknotting
- **Contact forces**
  - Self contact
  - Obstacle
  - Used to computer friction forces
- **Gravitational force**

$$\mathbf{f}_{\mathbf{g}} = Gm$$

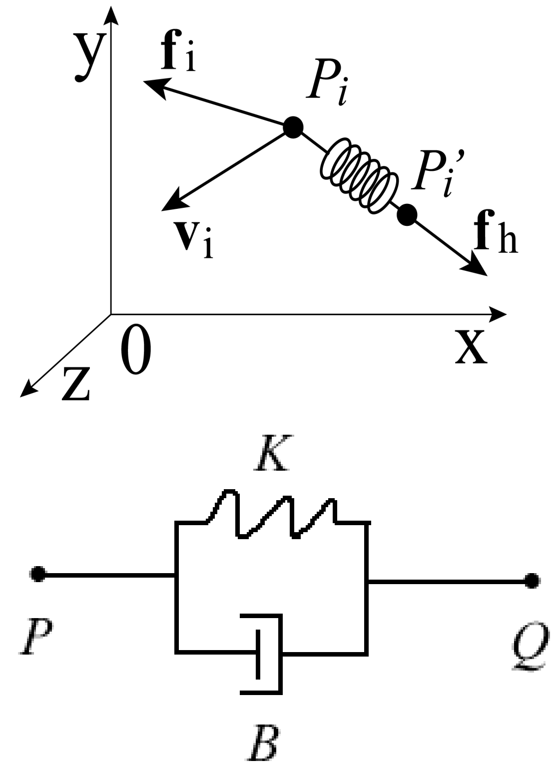
Where

$G = 9.8N/kg$

m - Mass of one mass point

User Input Force

- **Virtual coupling technique**
  - introduces a spring-damper between a simulated body and the device end effecter
  - we can use different constants for computing the output force for the device versus the input force for the simulated body



Virtual Coupling

Reference: J.E. Colgate, M.C. Stanley, J.M. Brown, ***Issues in the haptic display of tool use***, Intelligent Robots and Systems 95

Friction

- **Friction force**
  - Static Friction (not implemented in this project)
  - Kinematic Friction
  - Relative velocity of contact points

  Suppose **F** is a point along segment $\overrightarrow{P_a P_b}$

  E is a point along segment $\overrightarrow{P_c P_d}$

  Then, the velocities of point **E** and **F** are:

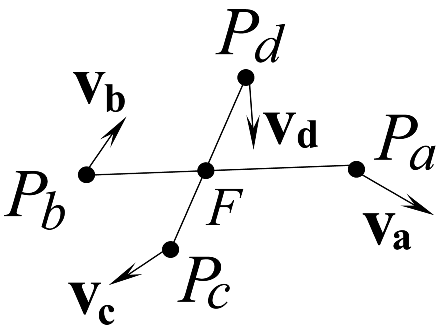
$$\mathbf{v_f} = (1-a)\mathbf{v_a} + a\mathbf{v_b}$$
$$\mathbf{v_e} = (1-b)\mathbf{v_c} + b\mathbf{v_d}$$

  a - the fraction of point **F** along segment $\overrightarrow{P_a P_b}$

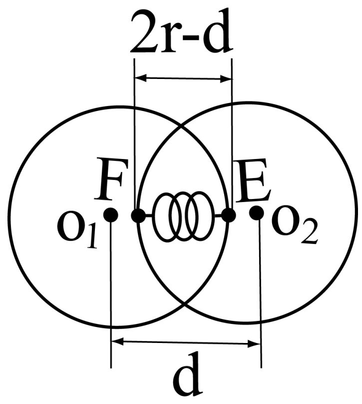
  b - the fraction of point **E** along segment $\overrightarrow{P_c P_d}$

  The relative velocity of point **E** and **F** is:

$$\mathbf{v_r} = \mathbf{v_f} - \mathbf{v_e}$$



Two sliding segments



Intersection of two segments in contact

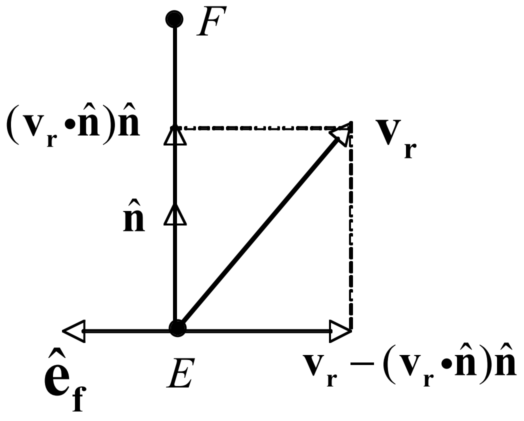
FRICTION – CONT.

- **Friction force**
  - Direction of Friction

$$\hat{\mathbf{e}}_\mathbf{f} = -\frac{\mathbf{v}_\mathbf{r} - (\mathbf{v}_\mathbf{r} \bullet \hat{\mathbf{n}})\hat{\mathbf{n}}}{\| \mathbf{v}_\mathbf{r} - (\mathbf{v}_\mathbf{r} \bullet \hat{\mathbf{n}})\hat{\mathbf{n}} \|}.$$

$\mathbf{v}_\mathbf{r}$ - Relative velocity

$\hat{\mathbf{n}}$ - Unit vector from point **E** to **F**



Direction of Friction

  - Normal Force

$$\mathbf{f}_\mathbf{n} = (k_{rs}(2r-d) - k_{rd}(\mathbf{v}_\mathbf{r} \bullet \hat{\mathbf{n}}))\hat{\mathbf{n}}$$

k_{rs} - Spring constant

r - Rope radius

k_{rd} - Damper constant

d - Distance between center lines of two segments

$\mathbf{v}_\mathbf{r}$ - Relative velocity

$\hat{\mathbf{n}}$ - Unit vector of normal force

Friction – cont.

- **Friction force**
  - Kinematic Friction

$$\mathbf{f_f} = \mu \, \| \mathbf{f_n} \| \, \hat{\mathbf{e}}_\mathbf{f}$$

Where

μ - Friction constant

$\mathbf{f_n}$ - Normal force

$\hat{\mathbf{e}}_\mathbf{f}$ - Friction direction vector

Internal Forces

- **Linear spring force**

$$\mathbf{f_s} = k_l \Delta l \hat{\mathbf{e}}_\mathbf{i}$$

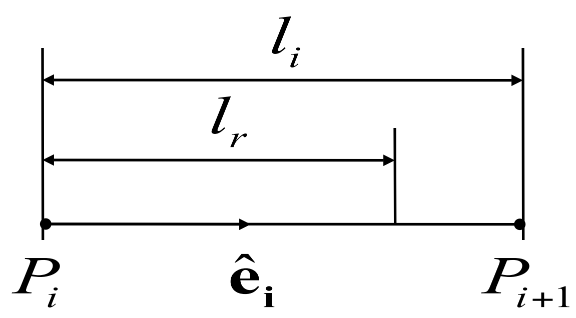
Where

$$\Delta l = \frac{l_i - l_r}{l_r}, \quad \hat{\mathbf{e}}_\mathbf{i} = \frac{P_{i+1} - P_i}{\| P_{i+1} - P_i \|}$$

l_i - Current length of rope segment

l_r - Rest length of rope segment

k_l - Linear spring constant



Linear Spring

- **Linear damper**

$$\mathbf{f_d} = k_d (v_{i+1} - v_i) \hat{\mathbf{e}}_\mathbf{i}$$

where

$$v_{i+1} = \mathbf{v_{i+1}} \cdot \hat{\mathbf{e}}_\mathbf{i}, \quad v_i = \mathbf{v_i} \cdot \hat{\mathbf{e}}_\mathbf{i}$$

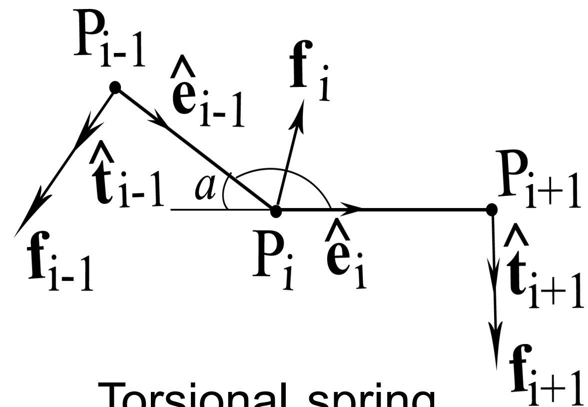
Internal Forces - cont.

- **Torsional spring**

$$\mathbf{f}_{\mathbf{i-1}} = k_{ts} \frac{\alpha}{\pi \, \| P_{i-1} - P_i \|} \hat{\mathbf{t}}_{\mathbf{i-1}}$$

$$\mathbf{f}_{\mathbf{i+1}} = k_{ts} \frac{\alpha}{\pi \, \| P_{i+1} - P_i \|} \hat{\mathbf{t}}_{\mathbf{i+1}}$$

$$\mathbf{f}_{\mathbf{i}} = -(\mathbf{f}_{\mathbf{i-1}} + \mathbf{f}_{\mathbf{i+1}})$$



Torsional spring

Where

$$\alpha = \begin{cases} \arcsin(\| \hat{\mathbf{e}}_{\mathbf{i-1}} \cdot \hat{\mathbf{e}}_{\mathbf{i}} \|) & \text{if} \quad \hat{\mathbf{e}}_{\mathbf{i-1}} \cdot \hat{\mathbf{e}}_{\mathbf{i}} > 0 \\ \pi - \arcsin(\| \hat{\mathbf{e}}_{\mathbf{i-1}} \cdot \hat{\mathbf{e}}_{\mathbf{i}} \|) & \text{if} \quad \hat{\mathbf{e}}_{\mathbf{i-1}} \cdot \hat{\mathbf{e}}_{\mathbf{i}} < 0 \end{cases}$$

$$\hat{\mathbf{t}}_{\mathbf{i+1}} = \hat{\mathbf{e}}_{\mathbf{i}} \times (\mathbf{e}_{\mathbf{i-1}} \times \mathbf{e}_{\mathbf{i}}), \qquad \hat{\mathbf{t}}_{\mathbf{i-1}} = \hat{\mathbf{e}}_{\mathbf{i-1}} \times (\mathbf{e}_{\mathbf{i-1}} \times \mathbf{e}_{\mathbf{i}})$$

$\hat{\mathbf{t}}_{\mathbf{i-1}}, \hat{\mathbf{t}}_{\mathbf{i+1}}$ - unit vectors of directions of $\mathbf{f}_{\mathbf{i-1}}, \mathbf{f}_{\mathbf{i+1}}$

Internal Forces - cont.

- **Torsional damper**

$$\mathbf{f_{i-1}} = k_{td}\left(\frac{(v_{i-1} - v_{ib})}{\| P_{i-1} - P_i \|} + \frac{(v_{i+1} - v_{ia})}{\| P_{i+1} - P_i \|}\right)\frac{\mathbf{\hat{t}_{i-1}}}{\| P_{i-1} - P_i \|}$$

$$\mathbf{f_{i+1}} = k_{td}\left(\frac{(v_{i-1} - v_{ib})}{\| P_{i-1} - P_i \|} + \frac{(v_{i+1} - v_{ia})}{\| P_{i+1} - P_i \|}\right)\frac{\mathbf{\hat{t}_{i+1}}}{\| P_{i+1} - P_i \|}$$

$$\mathbf{f_i} = -(\mathbf{f_{i-1}} + \mathbf{f_{i+1}})$$

where

k_{td} - Torsional damper constant

$$v_{i-1} = \mathbf{v_{i-1}} \cdot \mathbf{\hat{t}_{i-1}}, \quad v_{ib} = \mathbf{v_i} \cdot \mathbf{\hat{t}_{i-1}}, \quad v_{ia} = \mathbf{v_i} \cdot \mathbf{\hat{t}_{i+1}}, \quad v_{i+1} = \mathbf{v_{i+1}} \cdot \mathbf{\hat{t}_{i+1}}.$$

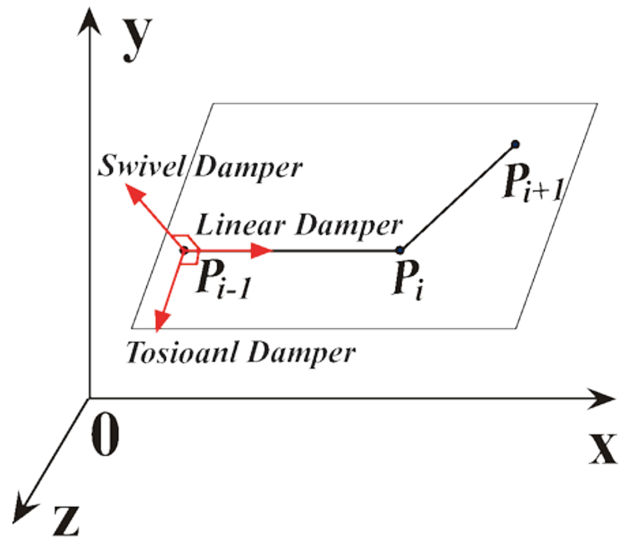
Internal Forces - cont.

- **Swivel damper**

$$\mathbf{f}_{\mathbf{i-1}} = k_{sw} \frac{(\mathbf{v}_{\mathbf{i-1}} - \mathbf{v}_{\mathbf{i}}) \cdot \hat{\mathbf{s}}}{\| P_{i-1} - P_i \|} \hat{\mathbf{s}}$$

$$\mathbf{f}_{\mathbf{i+1}} = k_{sw} \frac{(\mathbf{v}_{\mathbf{i+1}} - \mathbf{v}_{\mathbf{i}}) \cdot \hat{\mathbf{s}}}{\| P_{i+1} - P_i \|} \hat{\mathbf{s}}$$

$$\mathbf{f}_{\mathbf{i}} = -(\mathbf{f}_{\mathbf{i-1}} + \mathbf{f}_{\mathbf{i+1}})$$



Wher
e

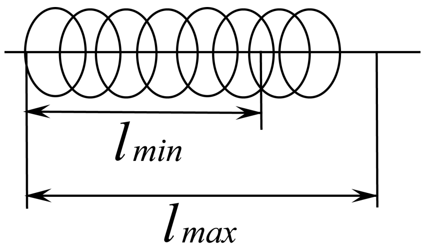
$$\hat{\mathbf{s}} = \mathbf{e}_{\mathbf{i}} \times \mathbf{e}_{i-1}$$

Force Propagation

- **Consideration:**
  Prevent suture from being stretched too long or compressed too short:

  $l_{\max}$ - Maximum length

  $l_{\min}$ - Minimum length

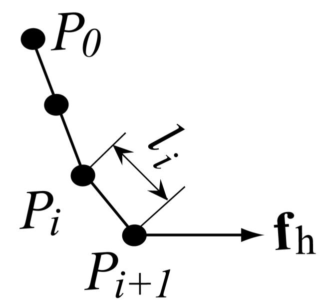


Suturing segment model

- **Condition A** – No propagation of user input force

  if $l_{\max} > l_i > l_{\min}$

  All the user input forces have been converted to internal forces



Pulling with one hand

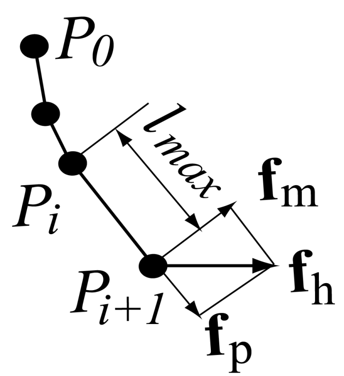
Force Propagation – cont.

- **Condition B**
  - Pulling with one hand

$$l_i = \begin{cases} l_{\max} & \text{if} \quad l_i' > l_{\max} \\ l_{\min} & \text{if} \quad l_i' < l_{\min} \end{cases}$$

where

l_i' - Expected length of the segment



Pulling with one hand

  - Force computation

$$\mathbf{f_p} = (\mathbf{f_h} \cdot \hat{\mathbf{e}}_\mathbf{i})\hat{\mathbf{e}}_\mathbf{i}$$
$$\mathbf{f}_m = (\mathbf{f_h} \cdot \hat{\mathbf{e}}_m)\hat{\mathbf{e}}_m$$

where

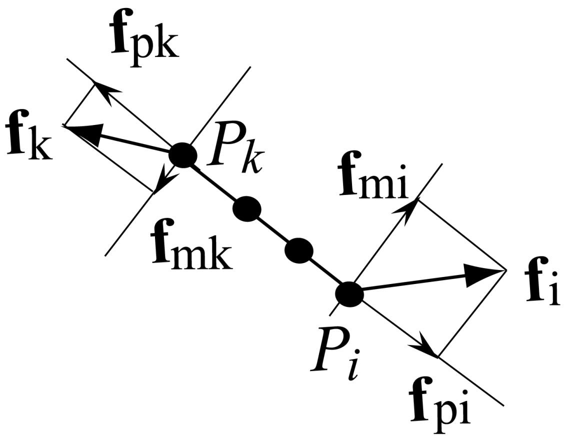
$\mathbf{f_h}$ - User input force, $\mathbf{f_p}$ - Force component which propagated,

$\mathbf{f_m}$ - Force component which creates motion

$\hat{\mathbf{e}}_i$ - the segment direction vector $\hat{\mathbf{e}}_m = \dfrac{\hat{\mathbf{e}}_i \times \mathbf{f}_h}{\|\hat{\mathbf{e}}_i \times \mathbf{f}_h\|} \times \hat{\mathbf{e}}_i.$

Force Propagation – cont.

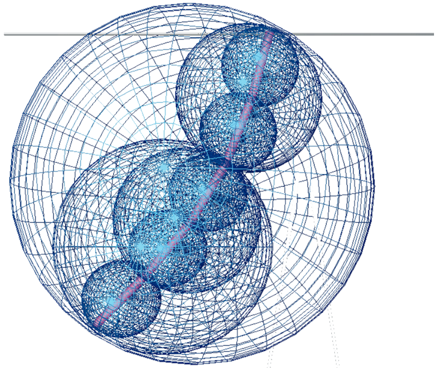
- **Condition C** (Pulling two points)
  - Force computation method is same as in condition B.
  - Starting from point P_i to P_k
  - Second compute from P_k to P_i



Pulling with two hands

Collision Detection and Management

- **Bounding-Volume Hierarchy(BVH)**
  - Self-collisions
  - Collision with other objects



BVH of a suture with five segments

- **Collision Management**
  - If segment distance $d < 2r$, each segment move away with $r - d/2 + \varepsilon/2$. Where ε is safety margin.
  - Velocity calculation – applying impulses to segment end points

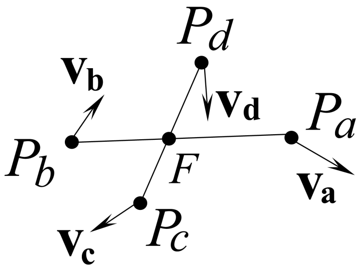
$$\mathbf{i} = \mathbf{f_n} \Delta t$$

Where

$\mathbf{i}$ - Impulse

$\mathbf{f_n}$ - Normal force

Δt - Time interval



Two sliding segments

Collision Management – cont.

– Collision Management

- Velocity calculation

$$i' = \frac{2\,\|\mathbf{i}\|}{a^2 + b^2 + (1-a)^2 + (1-b)^2},$$

$$\mathbf{v}_a^{new} = \mathbf{v}_a + (1-a)\frac{i'}{m}\hat{\mathbf{n}},$$

$$\mathbf{v}_b^{new} = \mathbf{v}_b + a\frac{i'}{m}\hat{\mathbf{n}},$$

$$\mathbf{v}_c^{new} = \mathbf{v}_c + (1-b)\frac{i'}{m}\hat{\mathbf{n}},$$

$$\mathbf{v}_d^{new} = \mathbf{v}_d + b\frac{i'}{m}\hat{\mathbf{n}},$$

Where
,

a - Fraction of point F along $\overrightarrow{P_a P_b}$

b - Fraction of point E along $\overrightarrow{P_c P_d}$

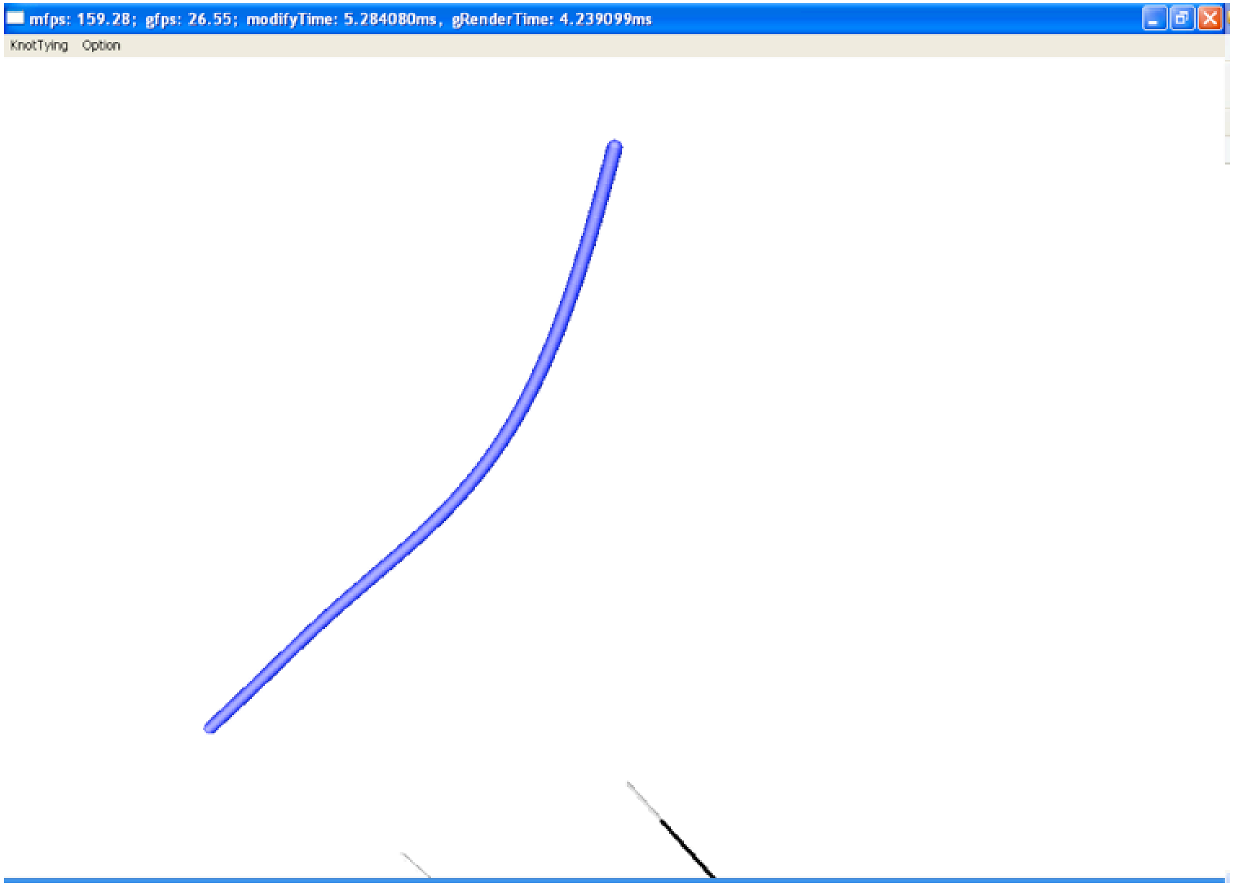
m - Mass of one point

$\hat{\mathbf{n}}$ - Unit vector of normal force

Reference: R. Bridson, R. Fedkiw, J. Anderson, ***Robust treatment of collisions, contact and friction for cloth animation*** 2002

Force Study

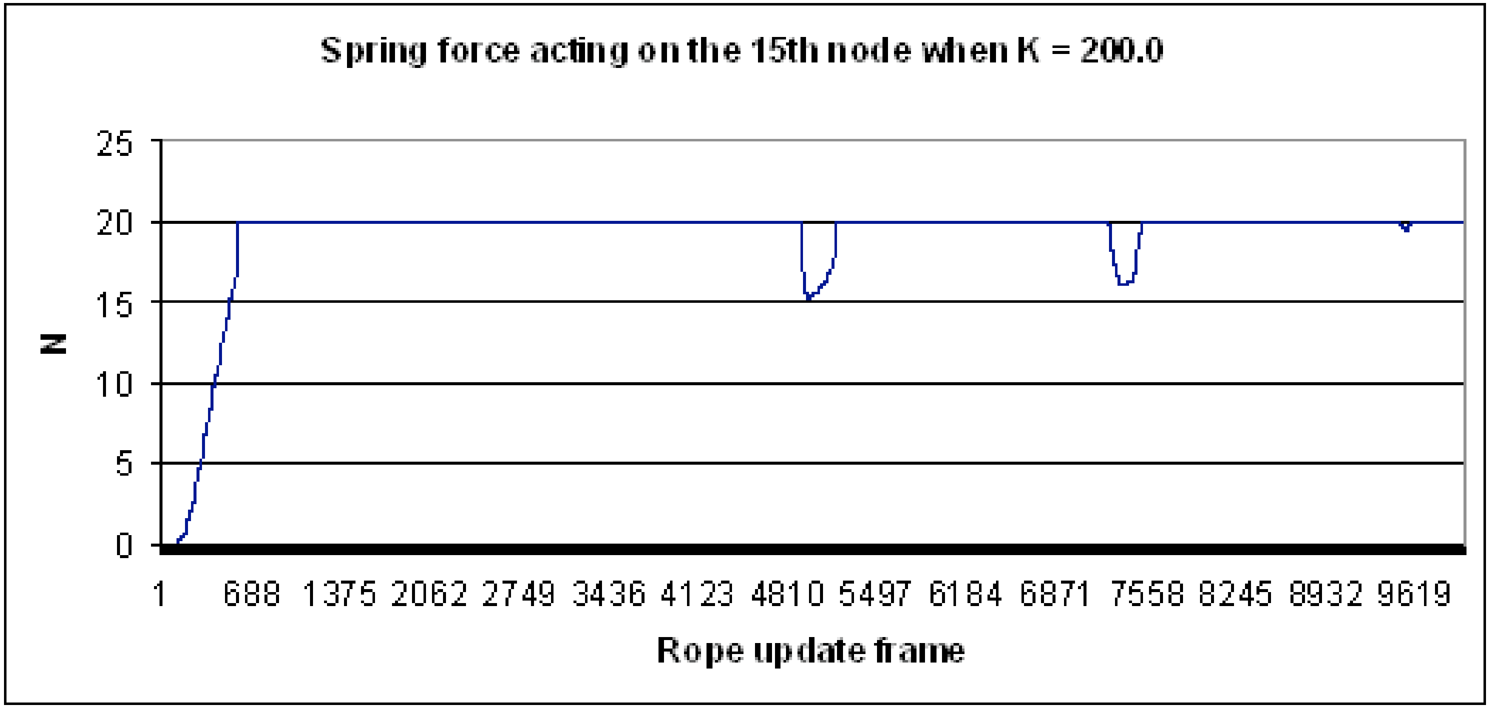
- **Condition - rope swings freely**



Force Study

- **Spring Force - acting on one node (15th in this example)**

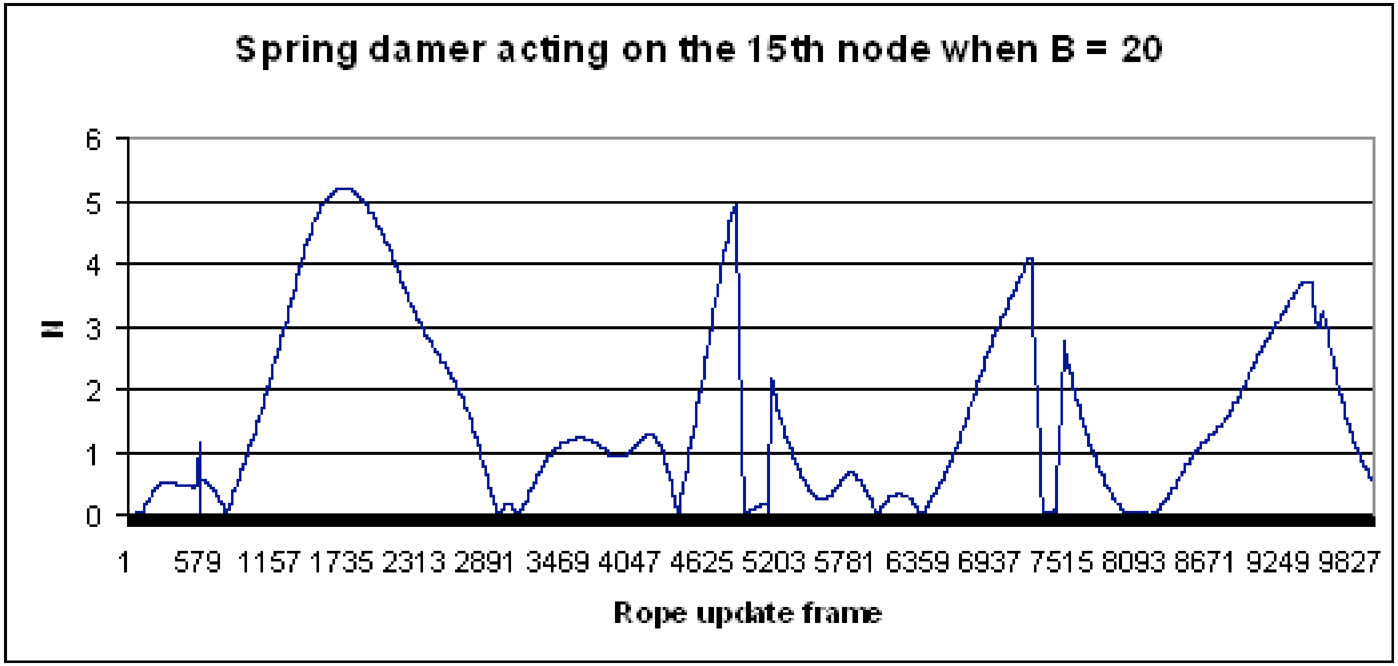
$$\mathbf{f_s} = k_l \Delta l \hat{\mathbf{e}}_\mathbf{i}$$



FORCE STUDY

- **Spring Damper Forces**

$$\mathbf{f_d} = k_d (v_{i+1} - v_i)\hat{\mathbf{e}}_\mathbf{i}$$



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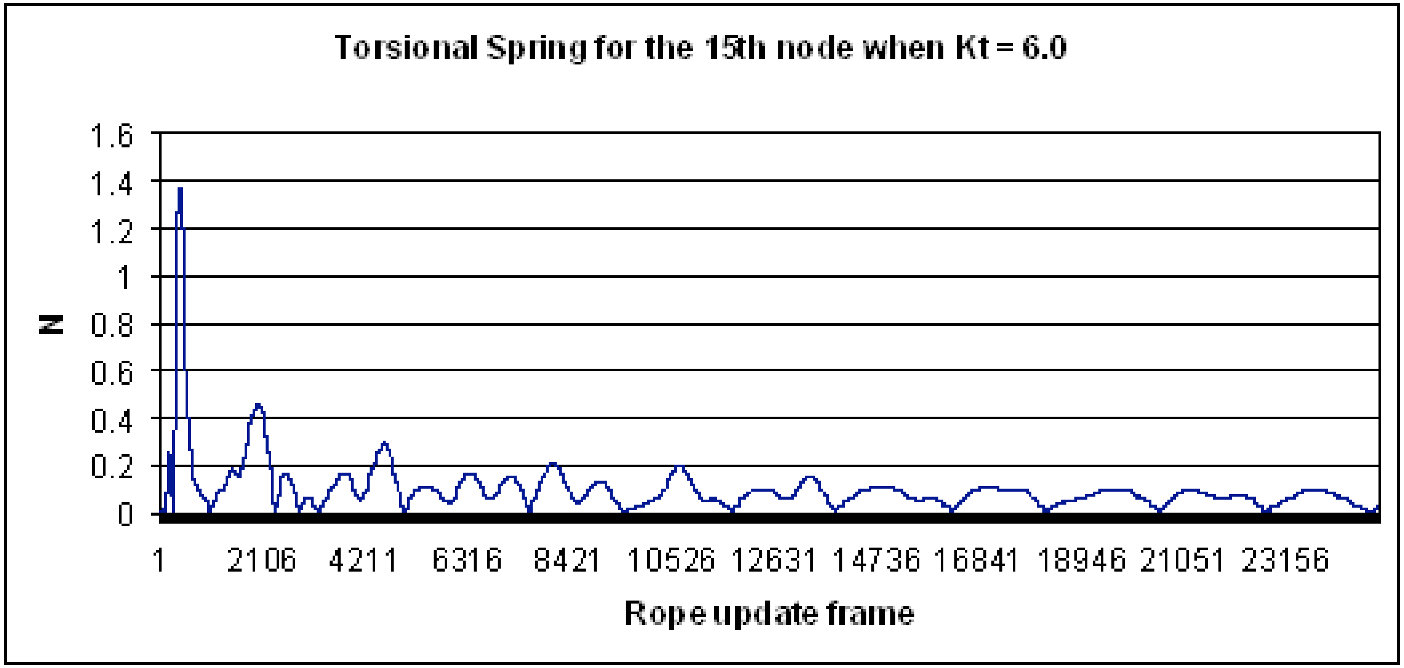
Force Study

- **Torsional Spring Forces**

$$\mathbf{f}_{\mathbf{i-1}} = k_{ts} \frac{\alpha}{\pi \, \| P_{i-1} - P_i \|} \hat{\mathbf{t}}_{\mathbf{i-1}}$$

$$\mathbf{f}_{\mathbf{i+1}} = k_{ts} \frac{\alpha}{\pi \, \| P_{i+1} - P_i \|} \hat{\mathbf{t}}_{\mathbf{i+1}}$$

$$\mathbf{f}_{\mathbf{i}} = -(\mathbf{f}_{\mathbf{i-1}} + \mathbf{f}_{\mathbf{i+1}})$$



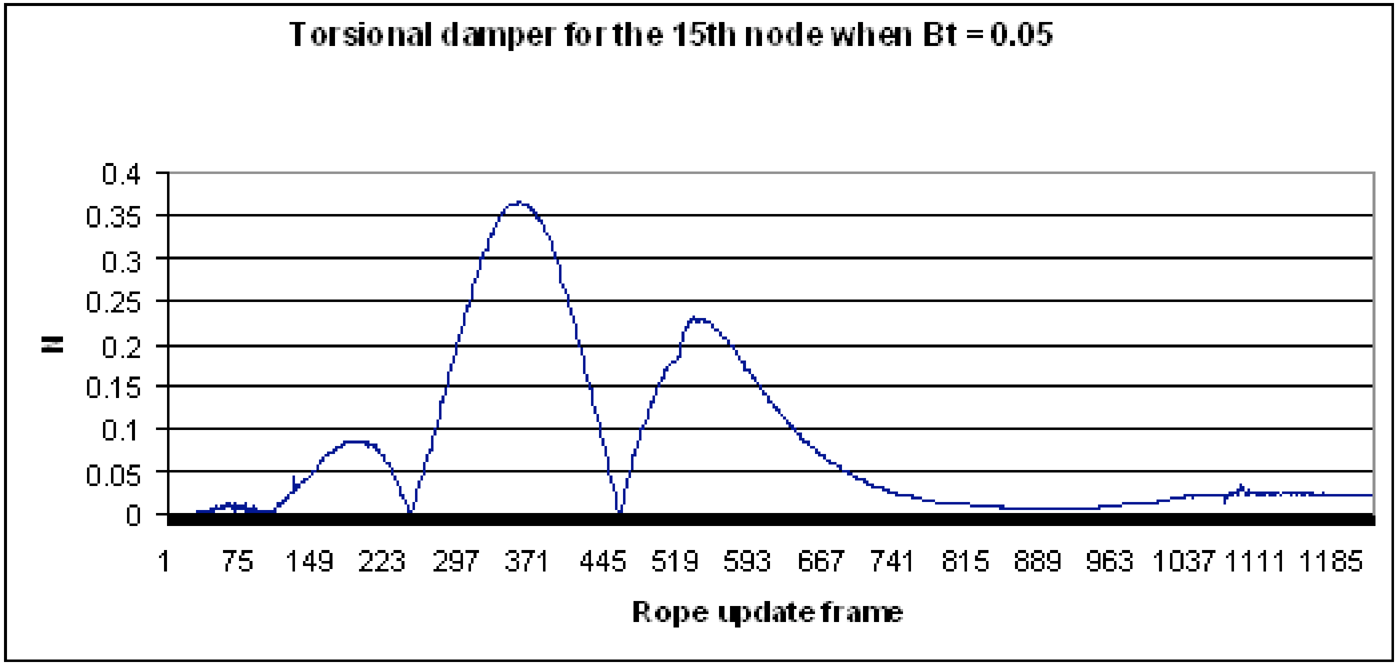
Force Study

- **Torsional Damper Forces**

$$\mathbf{f_{i\text{-}1}} = k_{td}\left(\frac{(v_{i-1} - v_{ib})}{\| P_{i-1} - P_i \|} + \frac{(v_{i+1} - v_{ia})}{\| P_{i+1} - P_i \|}\right)\frac{\hat{\mathbf{t}}_{\mathbf{i\text{-}1}}}{\| P_{i-1} - P_i \|}$$

$$\mathbf{f_{i+1}} = k_{td}\left(\frac{(v_{i-1} - v_{ib})}{\| P_{i-1} - P_i \|} + \frac{(v_{i+1} - v_{ia})}{\| P_{i+1} - P_i \|}\right)\frac{\hat{\mathbf{t}}_{\mathbf{i+1}}}{\| P_{i+1} - P_i \|}$$

$$\mathbf{f_i} = -(\mathbf{f_{i\text{-}1}} + \mathbf{f_{i+1}})$$



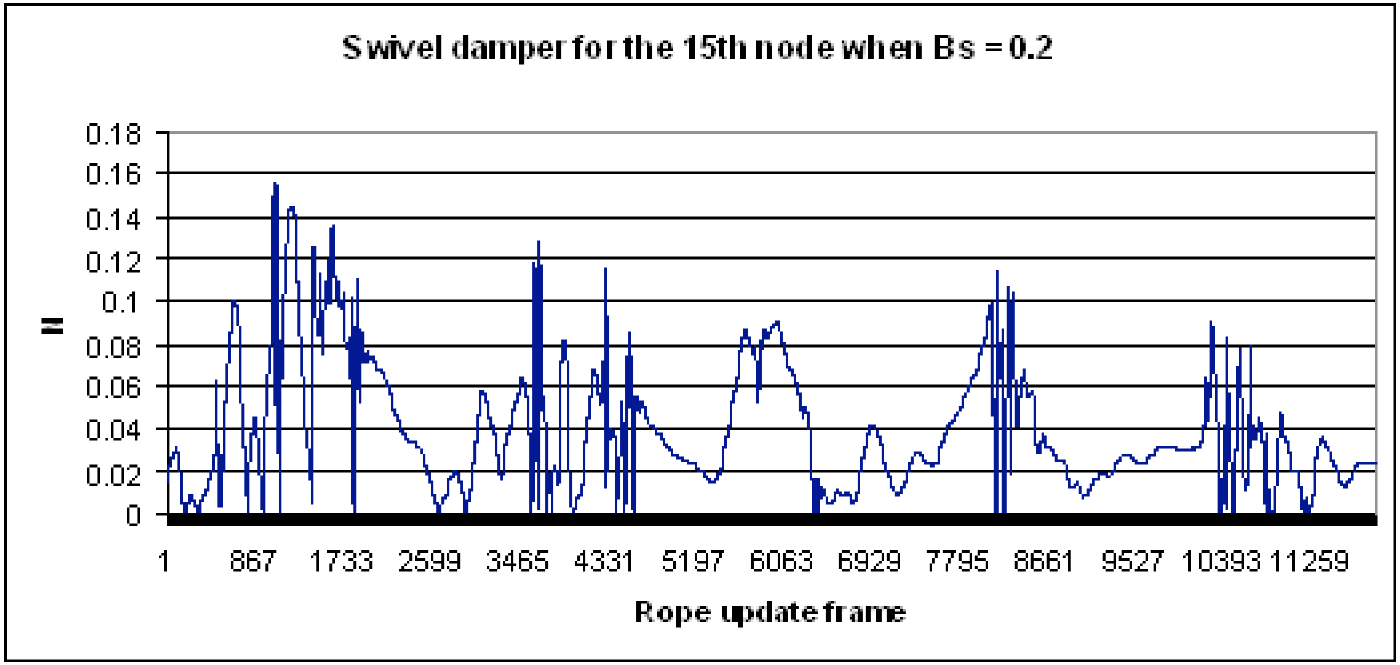
Force Study

- **Swivel Damper Forces**

$$\mathbf{f}_{\mathbf{i-1}} = k_{sw}\frac{(\mathbf{v}_{\mathbf{i-1}} - \mathbf{v}_{\mathbf{i}})\cdot\hat{\mathbf{s}}}{\| P_{i-1} - P_i \|}\hat{\mathbf{s}}$$

$$\mathbf{f}_{\mathbf{i+1}} = k_{sw}\frac{(\mathbf{v}_{\mathbf{i+1}} - \mathbf{v}_{\mathbf{i}})\cdot\hat{\mathbf{s}}}{\| P_{i+1} - P_i \|}\hat{\mathbf{s}}$$

$$\mathbf{f}_{\mathbf{i}} = -(\mathbf{f}_{\mathbf{i-1}} + \mathbf{f}_{\mathbf{i+1}})$$



Force Study

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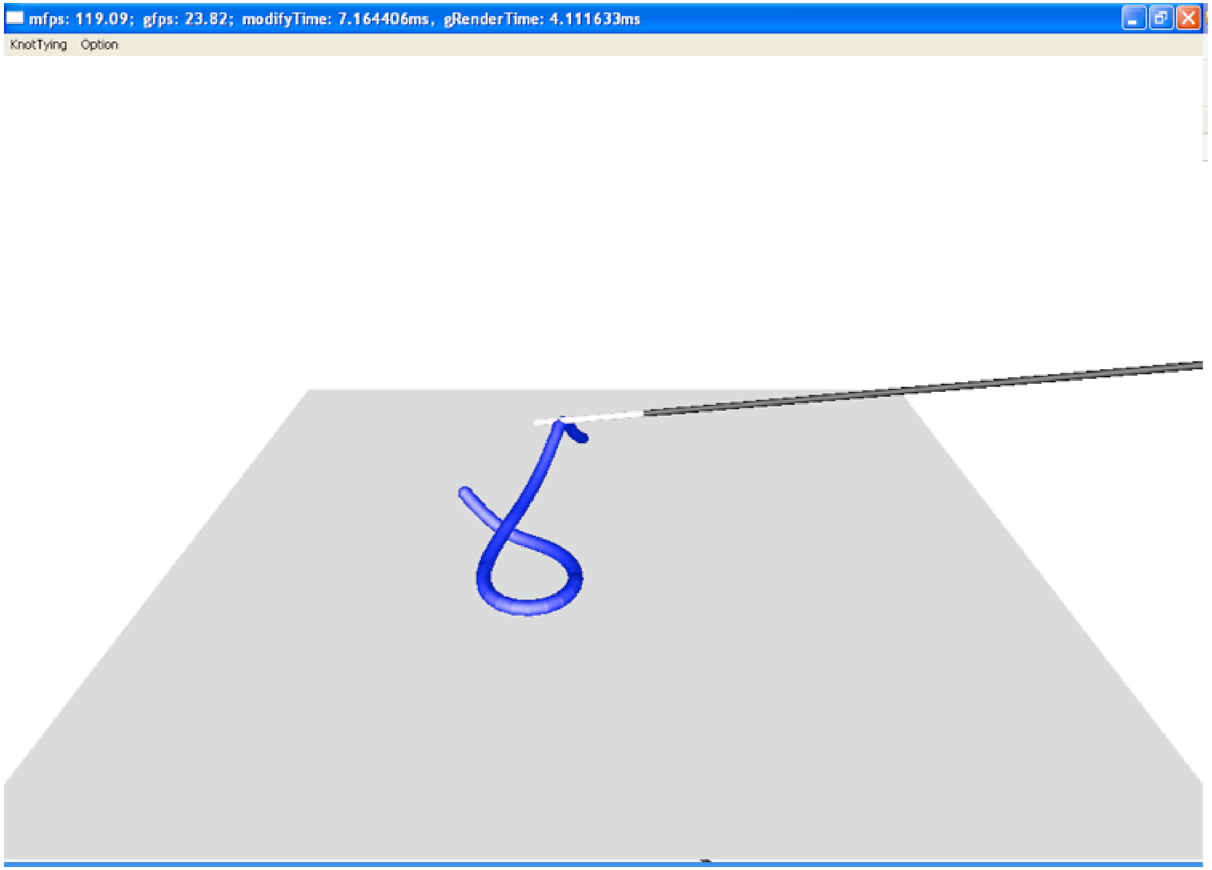
- **Friction - sliding over itself (one contact point)**

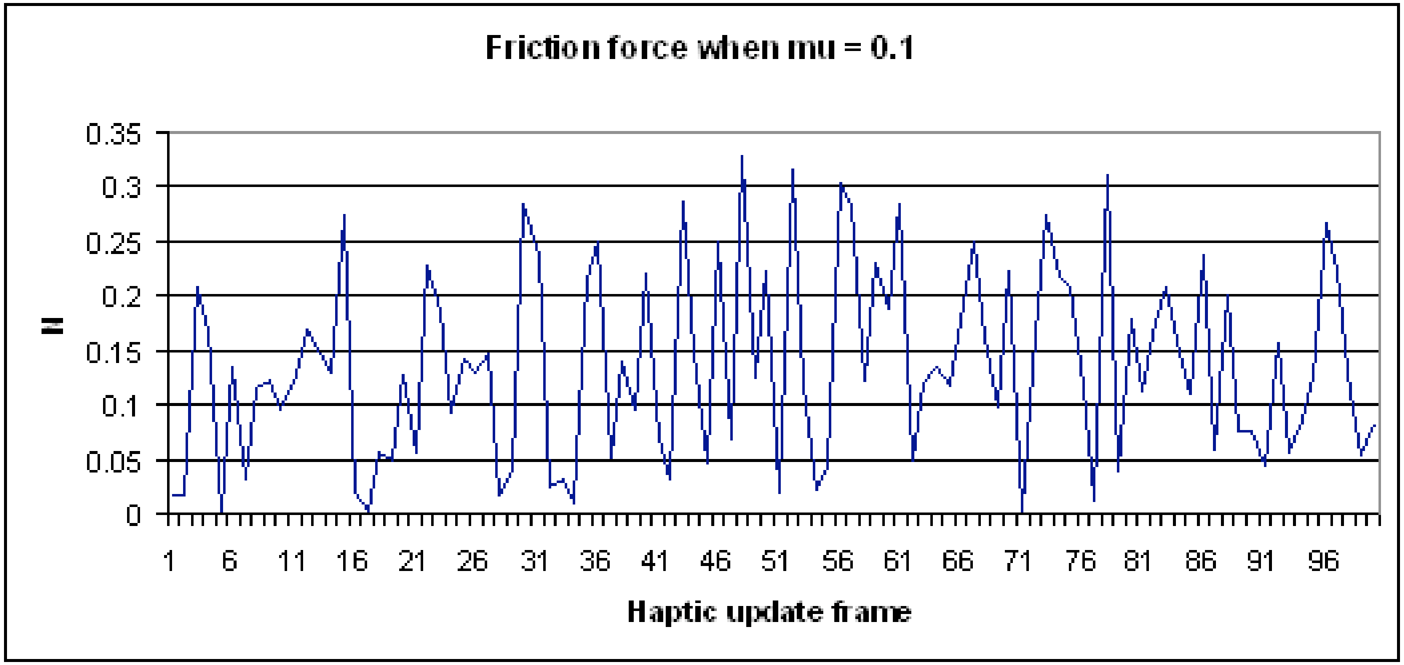


FORCE STUDY

- **Friction**

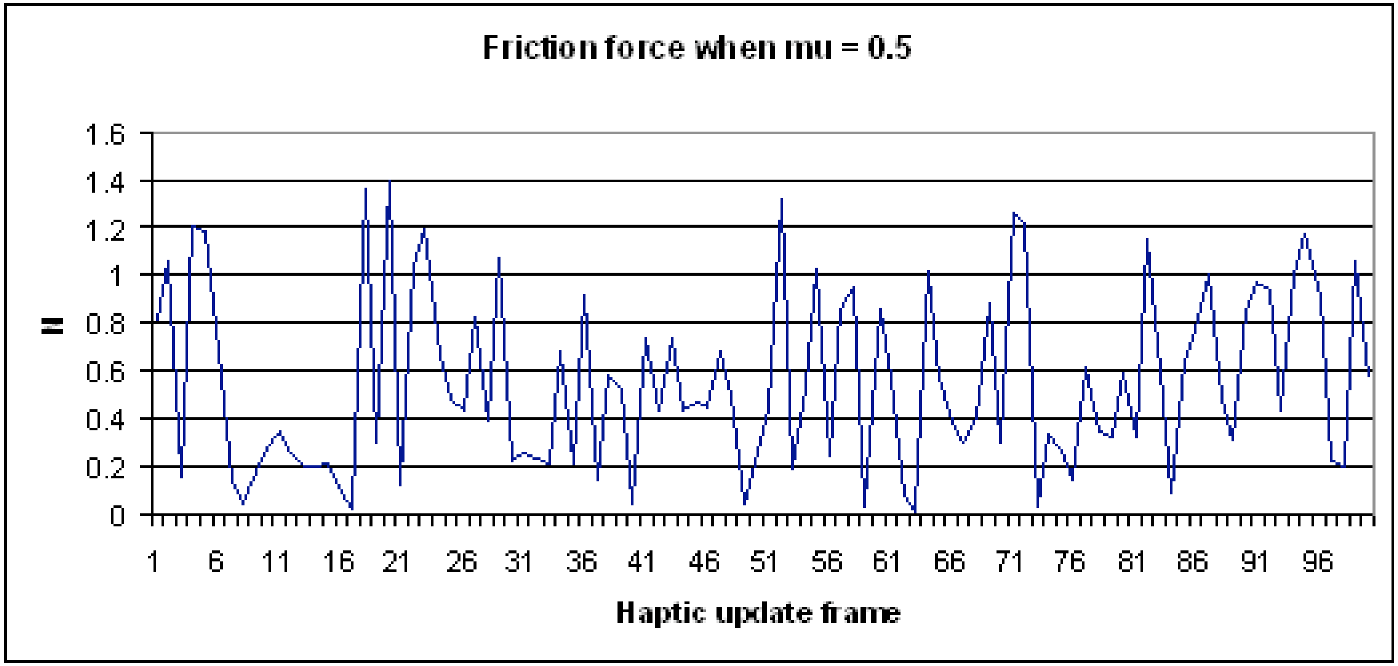
$$\mathbf{f_f} = \mu \,\|\, \mathbf{f_n} \,\|\, \hat{\mathbf{e}}_\mathbf{f}$$

$$\mathbf{f_n} = (k_{rs}(2r - d) - k_{rd}(\mathbf{v_r} \bullet \hat{\mathbf{n}}))\hat{\mathbf{n}} \qquad \hat{\mathbf{e}}_\mathbf{f} = -\frac{\mathbf{v_r} - (\mathbf{v_r} \bullet \hat{\mathbf{n}})\hat{\mathbf{n}}}{\| \mathbf{v_r} - (\mathbf{v_r} \bullet \hat{\mathbf{n}})\hat{\mathbf{n}} \|}.$$



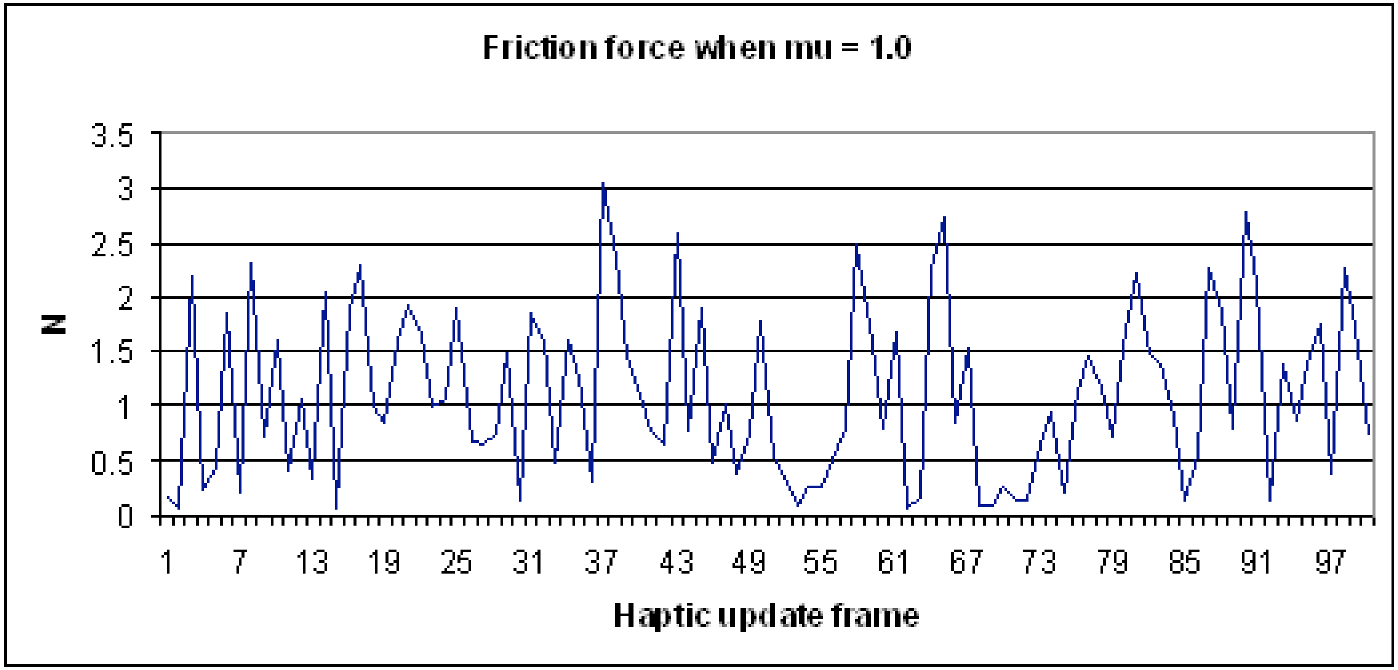
Force Study

- **Friction**



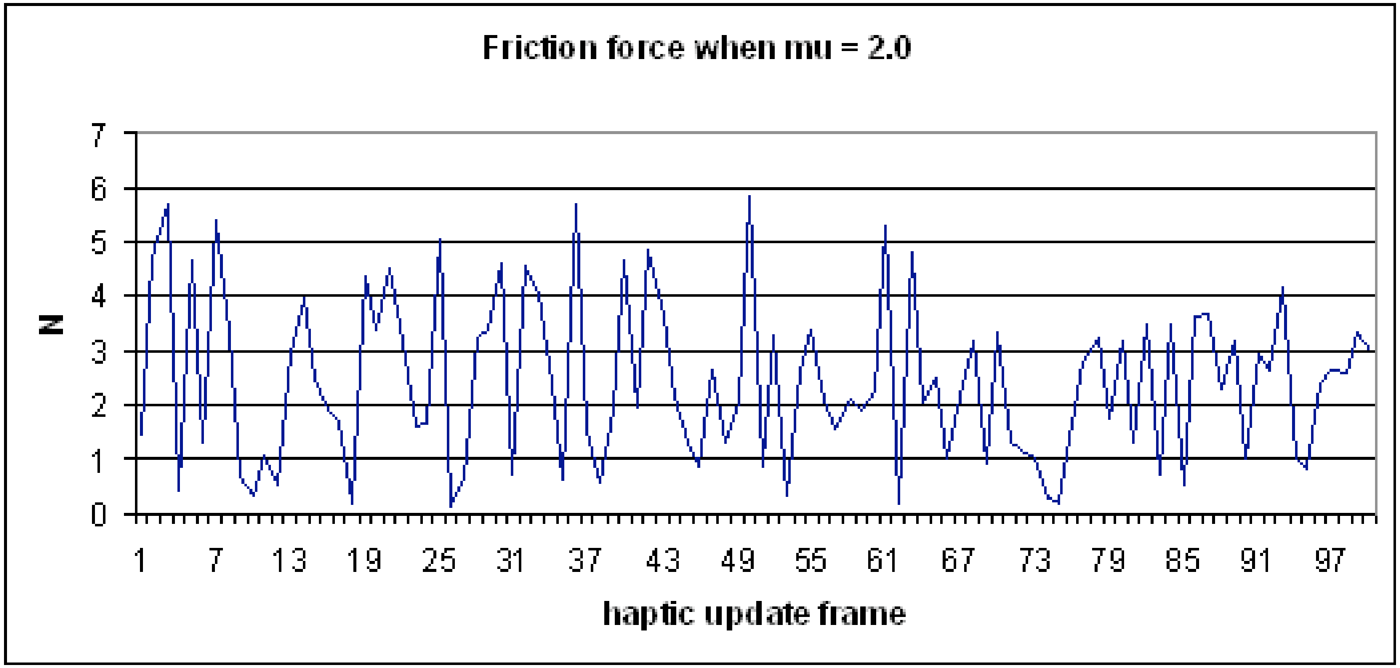
Force Study

- **Friction**



Force Study

- **Friction**



Case Study and Result – cont.

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- **Knotting Experiments**
  - We build several different models with various combinations of component forces
  - Model 1
    - Linear spring
    - Linear damper

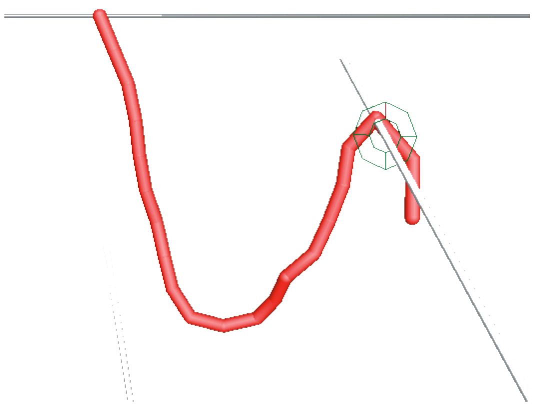


Fig.1 of Model 1

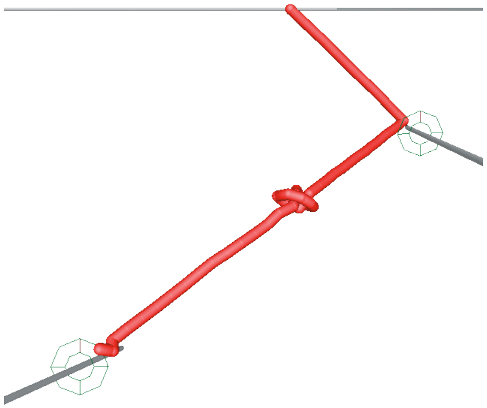


Fig.2 of Model 1

Knotting Experiments – cont.

– Model 2: Same as model 1 with added torsional spring

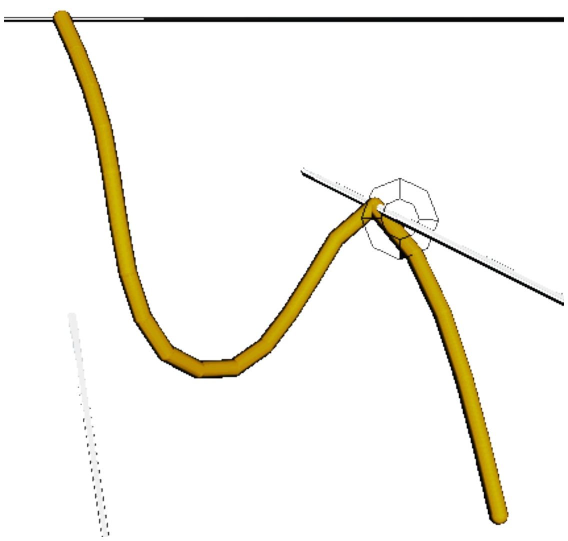


Fig.1 of Model 2

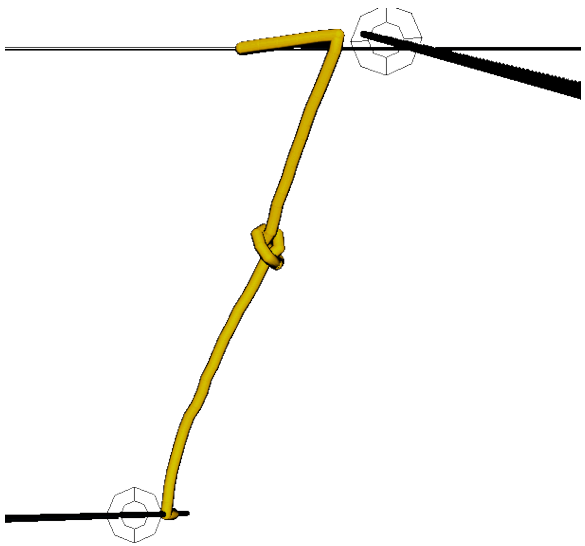


Fig.2 of Model 2

Knotting Experiments – cont.

– Model 3: Same as model 2 with added torsional damper

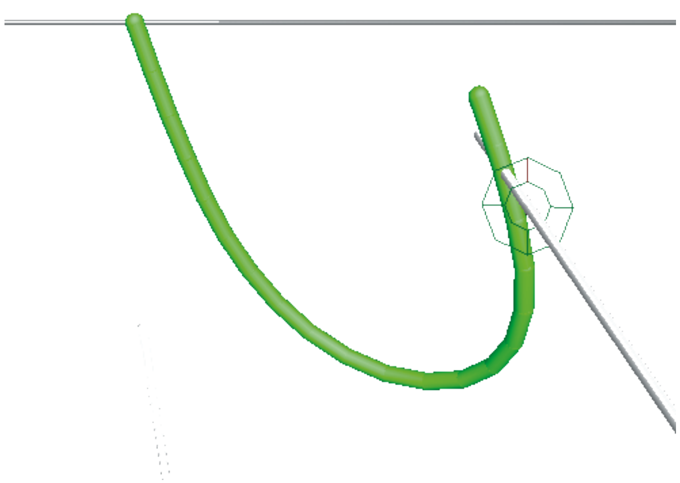


Fig.1 of Model 3

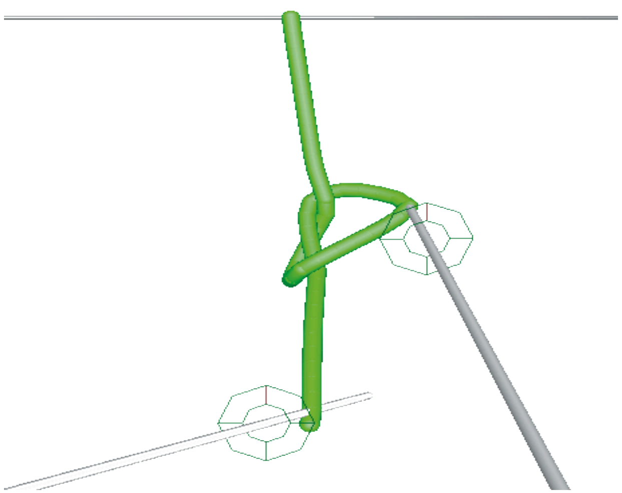


Fig.2 of Model 3

Knotting Experiments– cont.

– Model 4: Same as model 3 with added swivel damper

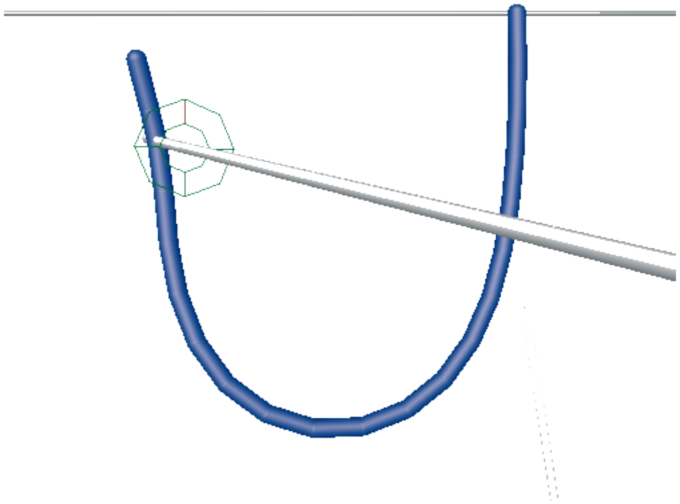


Fig.1 of Model 4

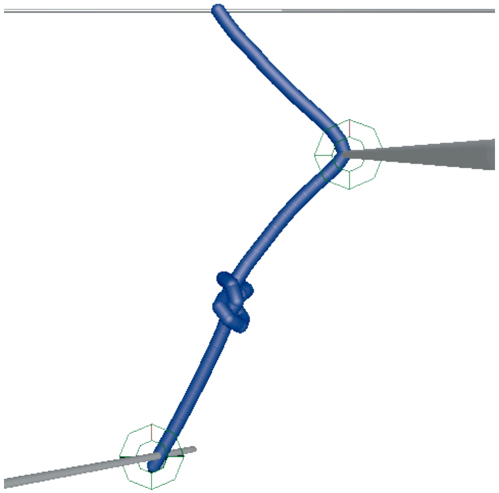
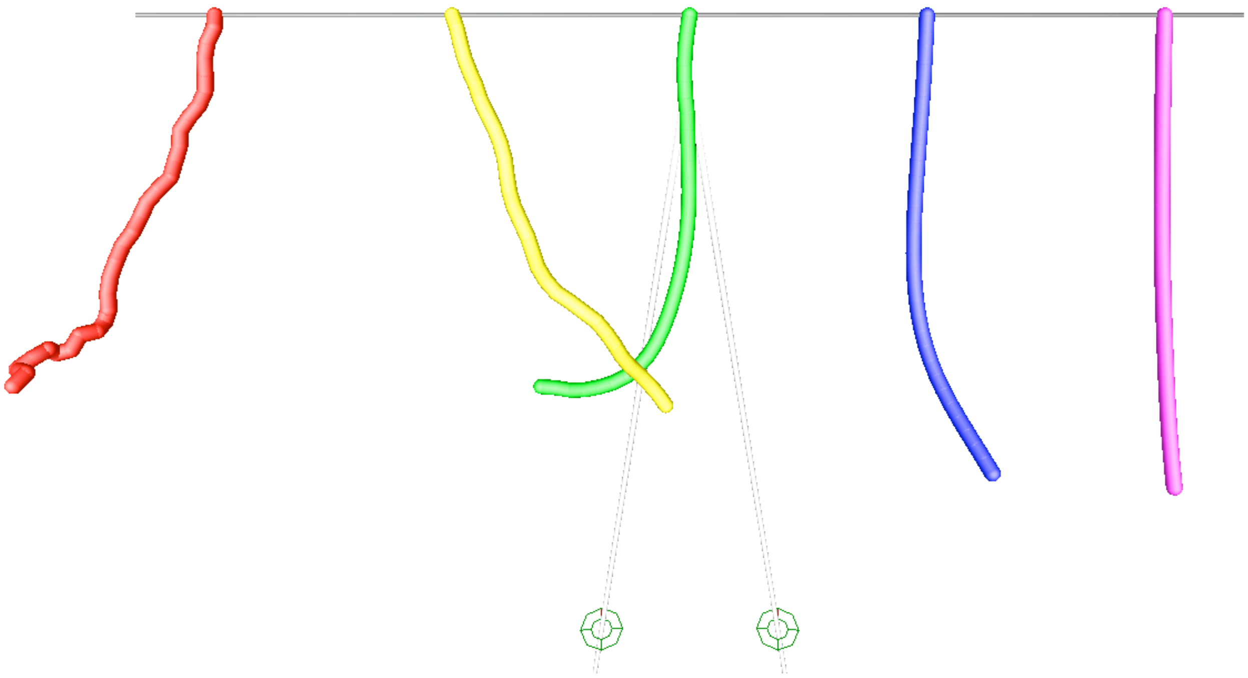


Fig.2 of Model 4

Knotting Experiments–

- Conclusion:
  - model 4 is the most realistic model
  - We take model 4 for the following experiments



Case Study and Result – cont.

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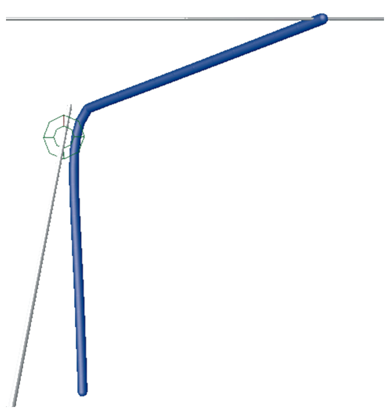
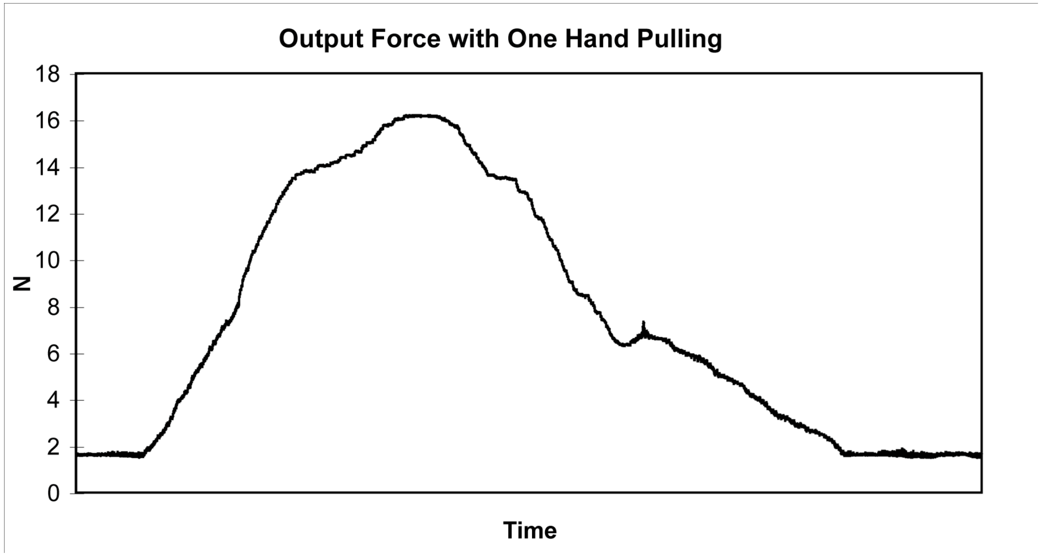
- **Force propagation Experiments**
  - Take model 4 as example
  - Plot the output forces which feed the haptic device in each haptic update frame during pulling
  - One Hand Pulling



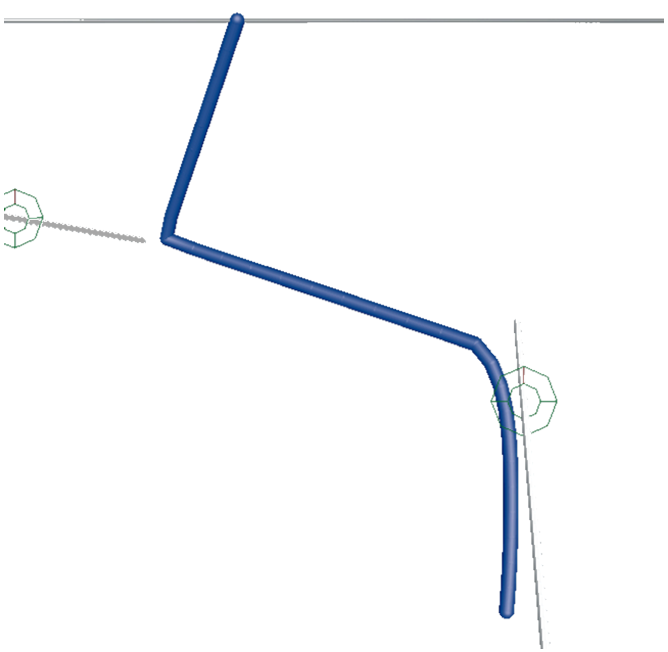
Screen shot of one hand pulling



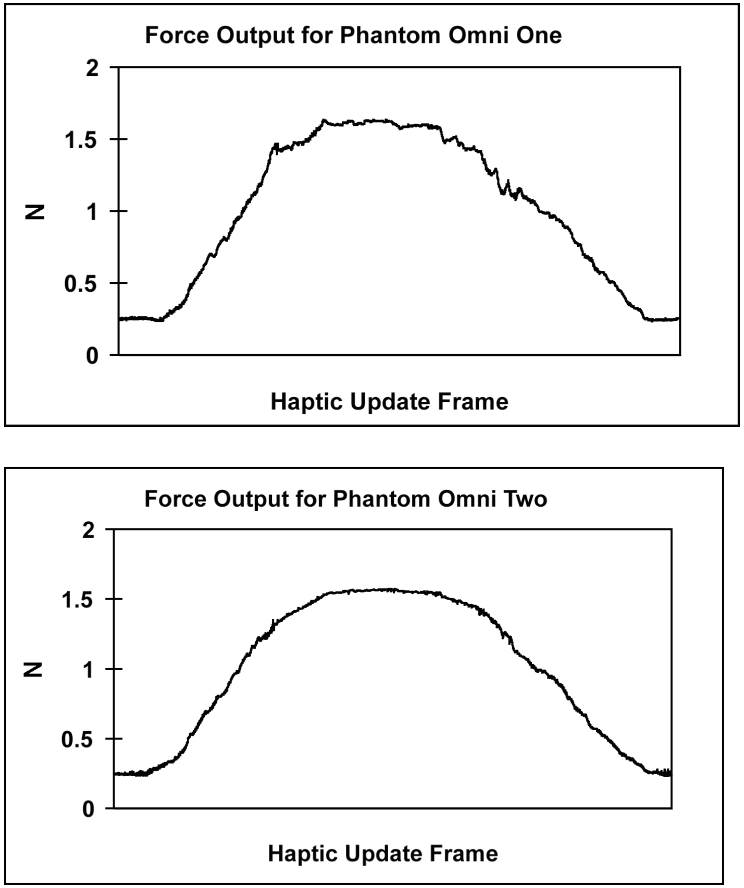
Force plot of one hand pulling

Force propagation Experiments— cont.

– Two-Hand Pulling



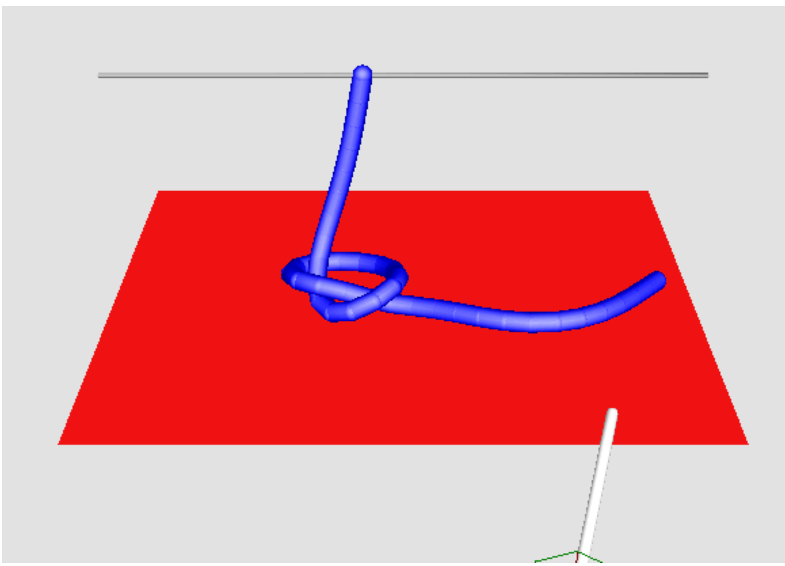
Screen shot of two-hand pulling



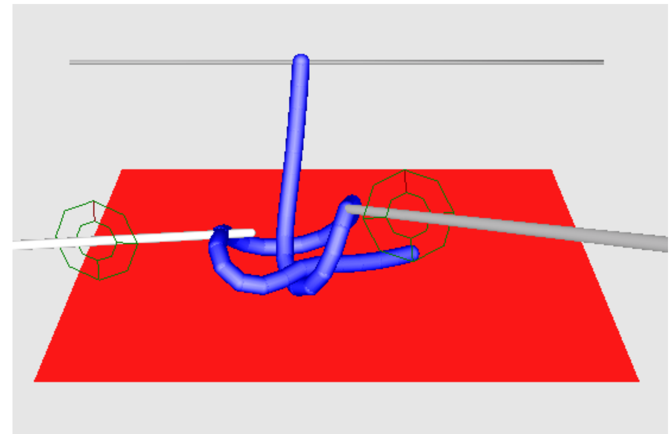
Force plot of two-hand pulling

Case Study and Result – cont.

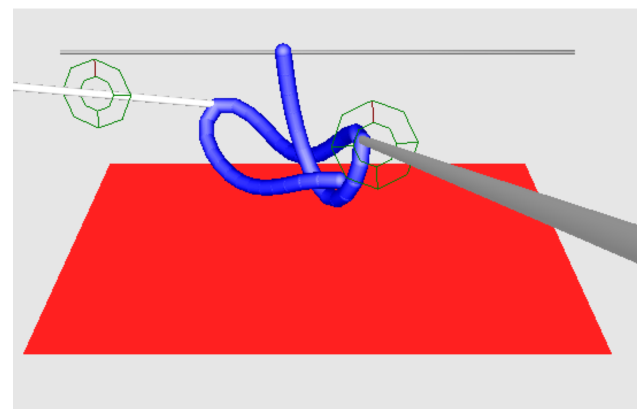
- **Unknotting Experiments**
  - Successful unknotting
    - Over hand knot



Over hand knot



Unknotting 1



Unknotting 2

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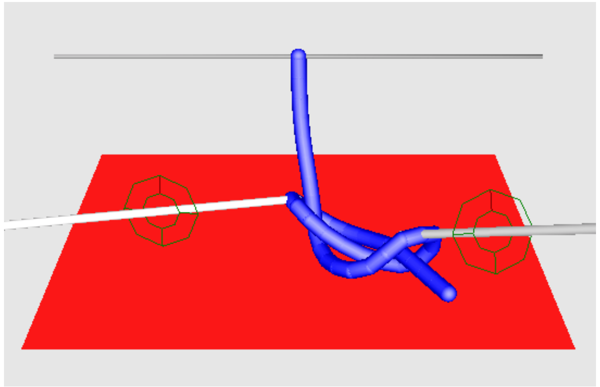
Unknotting Experiments – cont.

- Successful unknotting
  - Figure of eight knot



Unknotting 1

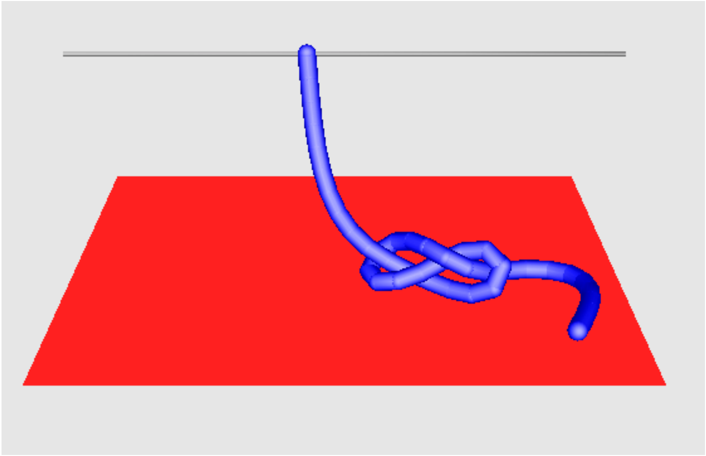
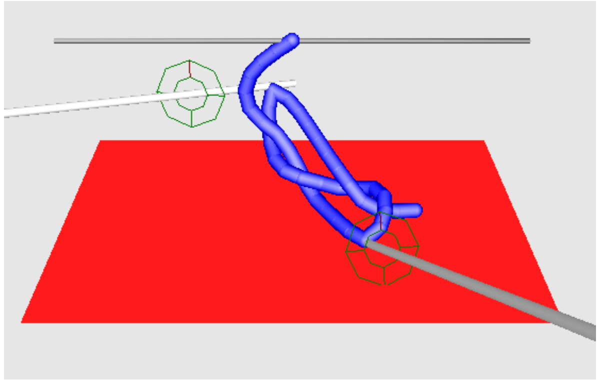


Figure of eight knot



Unknotting 1

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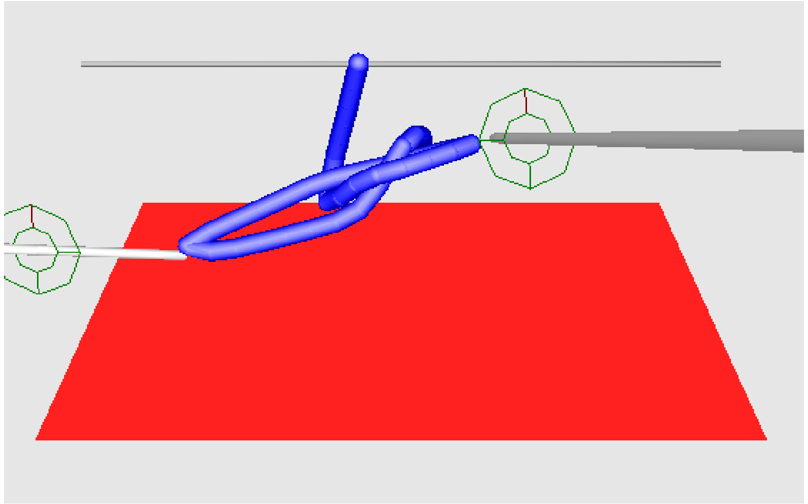
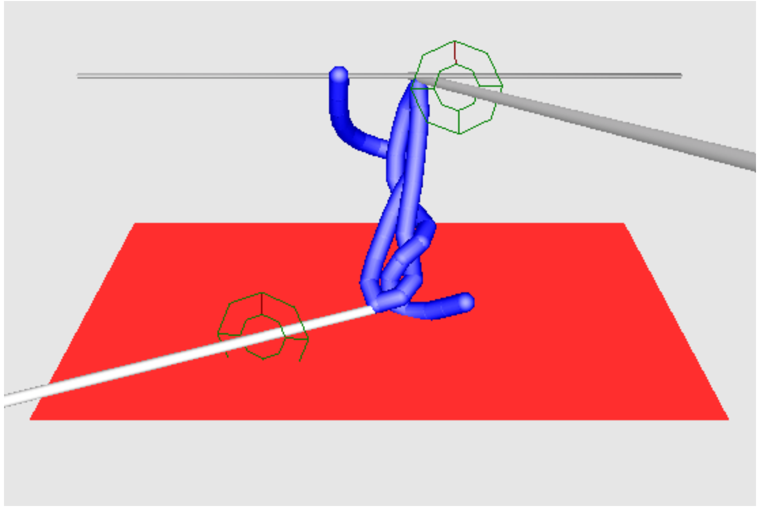
Unknotting Experiments – cont.

– Unsuccessful unknotting
  • Figure of eight knot



Unsuccessful unknotting 1



Unsuccessful unknotting 2

Thank you very much!

Any questions?